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Review

Applications of windowed Fourier fringe analysis in optical measurement: A review

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ABSTRACT

The applications of windowed Fourier fringe analysis in the past decade are reviewed. Because fringe patterns from different optical measurement systems are similar, the reviewed applications are classified according to the functions of the windowed Fourier transform being used in fringe pattern analysis: denosing exponential phase fields, demodulating carrier fringe patterns, getting phase derivatives, and utilizing local properties. From these applications, the windowed Fourier transform is shown to be effective and versatile for fringe pattern analysis.

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1. Introduction

After our first attempt [1], we found that windowed Fourier transform (WFT) was an effective and versatile tool for fringe pattern analysis [2–4]. We also realized later that the concept of "windowed" processing was important for fringe patterns, which has been summarized and emphasized in [5]. In fact, this concept was also important for other optical measurement methods such as digital image correlation [5]. Because we aimed at forming a theoretical framework of windowed fringe pattern analysis, we did not highlight applications in [5]. However, many successful and interesting applications of the WFT have emerged and have been demonstrated in the past decade. A review on these applications helps to elucidate how and where the WFT can be effectively applied, so as to extend the WFT to wider application areas in the future. The WFT is an extension of the well-established Fourier transform (FT) technique whose applications to metrology of extreme physical phenomena was recently reviewed [6].

We will soon observe that in these applications, fringe patterns from very different optical systems and application fields have been analyzed in similar ways. This is not surprising as the WFT is a data analysis tool and thus it concerns more on data features than data sources. The applications are thus classified into four categories according to the main functions of the WFT being used: denosing exponential phase fields, demodulating carrier fringe patterns, getting phase derivatives, and utilizing local properties. These four categories are reviewed from Section 3 to Section 6, respectively. Prior to the applications, the WFT will be briefly described in Section 2. The paper will be concluded in Section 7.

2. Windowed Fourier transform for fringe pattern analysis

In this section, fringe models, WFT based algorithms and their useful functions are briefly described to ease the review of applications.

2.1. Fringe models

A type of fringe patterns called exponential phase field (EPF) is of primary importance and can be written as

$$f(\boldsymbol{x}) = b(\boldsymbol{x})\exp[j\varphi(\boldsymbol{x})] + n(\boldsymbol{x}) \tag{1}$$

where $f(\boldsymbol{x})$, $b(\boldsymbol{x})$, $\varphi(\boldsymbol{x})$ and $n(\boldsymbol{x})$ are fringe value, fringe amplitude, phase distribution, and noise, respectively. The coordinate $\boldsymbol{x}$ can be a temporal coordinate t, a spatial coordinate x, a spatial coordinate y, or any combinations of them. In fringe analysis, $\boldsymbol{x} = t$ and $x = (\boldsymbol{x}, \boldsymbol{y})$ are most often used. In this section, $\boldsymbol{x} = x$ is used for explanation. More details can be found in [5].

The essential purpose of both the phase-shifting and the FT techniques is to construct EPFs from which the desired phase fields can be extracted. In the phase-shifting technique, an EPF can be directly obtained by combining phase-shifted fringe patterns. The FT technique processes a carrier fringe pattern which can be written as follows:

$$f(x) = a(x) + b(x)cos[\varphi(x) + \omega_{cx}x] + n(x), \tag{2}$$

where $a(x)$ and ω_{cx} are background intensity and carrier frequency, respectively. The carrier fringe pattern can be re-written as

$$f(x) = a(x) + c(x)exp(j\omega_{cx}x) + c^*(x)exp(-j\omega_{cx}x) + n(x) \tag{3}$$

where $c(x) = \frac{1}{2}b(x)exp[j\varphi(x)]$. There are four terms on the right hand of Eq. (3). If the carrier frequency ω_{cx} is high enough, the first three components are separable in the Fourier domain, while the noise usually permeates the entire domain. The FT technique selects the spectrum of the second component, shifts it to the origin, and then inverse transforms it to the spatial domain to recover $c(x)$, which is an EPF.

2.2. Algorithms based on windowed Fourier transform

The basis function of the WFT is

$$g_{\xi_x}(x) = g(x)\exp(j\xi_x x), \tag{4}$$

where $g(x)$ is a window function and ξ_x carries the meaning of frequency. The window function makes the frequency local. Because of the obvious similarity between the WFT basis function and EPFs, the WFT was selected as an analysis tool. The forward and inverse WFT can be written as

$$Sf(u;\ \xi_x) = f \otimes g_{\xi_x}(u), \tag{5}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sf(x;\ \xi_x) \otimes g_{\xi_x}(x)d\xi_x, \tag{6}$$

where $Sf(u;\xi_x)$ is the WFT spectrum and $\otimes$ denotes convolution. Two algorithms based on the WFT have been developed: the windowed Fourier ridges (WFR) and the windowed Fourier filtering (WFF).

The WFR highlights the idea of matching and searches the WFT basis function most similar to a local EPF, which can be formulated as

$$\hat{\omega}_x(u) = \arg\max_{\xi_x} |Sf(u;\ \xi_x)|, \tag{7}$$

$$\hat{\varphi}_w(u) = \angle Sf[u; \hat{\omega}_x(u)] - \frac{1}{2}\arctan[\sigma_x^2 \hat{c}_{xx}(u)], \tag{8}$$

where ˆ indicates the estimation of a parameter, $\omega_x(u) = d\varphi(u)/dx$ is the phase derivative and often called local frequency and $c_{xx}(u) = d^2\varphi(u)/dx^2$ is the second order phase derivative and often called local curvature. We have assumed that the window function is Gaussian and the standard deviation σ_x is used to indicate the window size. It is important to note that the second term on the right hand of Eq. (8) is used to correct the phase bias, which is important for accurate phase extraction.

The WFF highlights the idea of filtering and suppresses the noise by selecting a suitable frequency band containing signals and by thresholding to remove noise coefficients, which can be formulated as

$$\overline{f}(x) = \frac{\xi_{xi}}{2\pi}\sum_{\xi_x = \omega_{xl}}^{\omega_{xh}} \overline{Sf}(x;\xi_x) \otimes g_{\xi_x}(x), \tag{9}$$

$$\overline{Sf}(u;\xi_x) = \begin{cases} Sf(u;\ \xi_x) & if \quad |Sf(u;\ \xi_x)| \geq thr \\ 0 & if \quad |Sf(u;\xi_x)| < thr \end{cases}, \tag{10}$$

$$\overline{\varphi}_w(x) = \angle\overline{f}(x), \tag{11}$$