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Gradient domain methods with application to 4D scene reconstruction

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ABSTRACT

In many applications such as Photometric Stereo, Shape from Shading, Differential 3D reconstruction and Image Editing in gradient domain it is important to integrate a retrieved gradient field. In most of the real experiments, the retrieved gradient fields correspond to nonintegrable fields (i.e. they are not irrotational on every point of the domain). Robust approaches have been proposed to deal with noisy nonintegrable gradient fields. In this work we extend some of these techniques for the case of dynamic scenes when the gradient field in the $x-y$ domain can be estimated over time. We exploit temporal consistency in the scene to ensure integrability and improve the accuracy of the results. In addition, two known integration algorithms are reviewed and important implementation details are discussed. Experiments with synthetic and real data showing some potential applications for the proposed framework are presented.

1. Introduction

In many applications such as Photometric Stereo [1], Shape from Shading [2], Differential 3D reconstruction [3] and Image Editing in gradient domain [4,5] it is important to integrate a retrieved gradient field. The main problem when dealing with estimated gradient fields is that due to empirical errors and noise, these gradient fields are not usually integrable i.e. they do not represent irrotational fields. Classical approaches are focus on least squares solutions [6,7], or calculate the orthogonal projection onto a vector subspace defined by a finite set of basis functions to enforce integrability [8]. These techniques lack of robustness [9] and they are not suitable methods for data with large noise and outliers. Recently, more complex and robust approaches have been proposed; Agrawal and Raskar proposed a purely algebraic approach, they first showed that enforcing integrability can be formulated as solving a single linear system $Ax=b$ over the image, they showed conditions under which the system can be solved and a method to get to those conditions based on graph theory [10]. Vogel et al. presented a method based on homogeneous higher order regularization, thus it becomes possible to estimate the surface depth directly by solving a single partial differential equation [11]. Agrawal et al. [12] also proposed the use of spatially varying anisotropic weights, to achieve significant improvement in reconstructions, they propose (i) α-surfaces using binary weights, where the parameter α allows trade off between smoothness and robustness, (ii) M-estimators and edge preserving regularization using continuous weights and (iii) Diffusion using affine transformation of gradients. More recently, Ng et al. proposed the use of kernel basis functions, which transfer the continuous surface reconstruction problem into high-dimensional space, where a closed-form solution exists [13]. For a detailed description of some of these algorithms we recommend Agrawal's thesis [9] which is an excellent survey of surface reconstruction techniques from gradients.

Most of the integration algorithms were developed for two dimensional problems where one knows the estimations of the x and y partial derivatives of an unknown function $z(x,y)$. To the best of our knowledge none of them were extended to deal with dynamic scenes (i.e. $z(x,y,\mathbf{t})$). In this work we present a generalization for some well known integration algorithms to the case of dynamic scenes and we exploit the consistency in the movement of the scene to guide the integration process and make the algorithms more robust to random noise. The work that most closely resembles to ours is [14] where a new framework for spatio-temporal video editing in gradient domain is presented. In that case, it was assumed that the estimation of the x, y and t partial derivatives was available and that information was used to filter and process videos in the x, y and t gradient space. It is important to highlight that, in contrast to the problem treated in [14], we deal with problems in which we only have estimations of x and y partial derivatives (e.g. Differential 3D reconstruction and Photometric Stereo) and no information about the temporal partial derivative is available.

Section 2 covers the theoretical aspects of the presented technique, reviews some known integration methods and discusses some

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implementation details. Section 3 presents some experimental results: one synthetic example, a video from a public repository and a retrieved gradient field are processed and analyzed. Finally, Section 4 concludes the work and provides some possible future lines of research.

2. Variational formulation

In the following, we will assume that it is possible to estimate x and y partial derivatives ($g_x(x,y,t)$ and $g_y(x,y,t)$ respectively) for some unknown function $z:\Omega\times[0,T]\rightarrow\mathbb{R}$ where Ω is an open subset of $\mathbb{R}^2$. The estimated g_x and g_y partial derivatives of z can be obtained from different methods depending on the field and context. For example, for depth retrieval we recently presented a novel technique capable of providing the depth spatial partial derivatives over the time for a dynamic scene [3].

We can assume that

$$g_x(x,y,t)\approx z_x(x,y,t) \quad \text{and} \quad g_y(x,y,t)\approx z_y(x,y,t) \tag{1}$$

but the equality usually does not hold because of noise and surface singularities. In particular, this implies that in most of the cases $g_x(x,y,t)$ and $g_y(x,y,t)$ represent a not-integrable gradient field, i.e.[1]

$$\nabla_{xy}\times(g_x,g_y)\neq 0 \tag{2}$$

for some x, y and t. To enforce integrability, robust approaches must be considered. One of the simplest and oldest methods consists in minimizing the energy function:

$$E[u]=\int_\Omega\left(|u_x-g_x|^2+|u_y-g_y|^2\right)\,dx\,dy \tag{3}$$

whose Euler–Lagrange equation leads to the Poisson equation

$$\nabla^2_{xy}u=\mathrm{div}_{xy}(g_x,g_y). \tag{4}$$

Eq. (4) can be solved very fast and efficiently e.g. using Cosine or Fourier transformations (see [5] and references therein) for each time t independently.

The previous approach is a straightforward manner to solve the integration problem in a Least Squares (LS) sense, and for many applications represents the simplest solution to the problem. However, the LS solution lacks robustness and the result is significantly affected when data has noise and outliers [9]. Because of the simplicity and efficiency of LS methods, they may be tried always first. For smooth surfaces or noiseless experiments it may be enough to achieve an accurate solution. If borders are not preserved or surface details are lost after reconstruction, then it is worth to follow a more complex anisotropic approach.

We will also focus on a more robust method that consists in integrating an affine transformation of Gradients using a Diffusion Tensor [9]. This technique is based on Anisotropic Diffusion (AD) ideas proposed in the context of image restoration [15] and multiscale image analysis [16]. We will focus on the ideas proposed by Weickert [17,18] and we will adapt the general framework to the integration problem. Instead of solving the Poisson equation given by Eq. (4), the differential equation

$$\mathrm{div}_{xy}(D\nabla_{xy}u)=\mathrm{div}_{xy}(D(g_x,g_y)) \tag{5}$$

is considered. D represents an affine transformation (2×2 symmetric matrix for each x, y and t). The equation above can be thought as the Euler–Lagrange equation of the following energy function:

$$E[u]=\int_\Omega(d_{11}(u_x-g_x)^2+(d_{12}+d_{21})(u_x-g_x)(u_y-g_y)+d_{22}(u_y-g_y)^2)\,dx\,dy \tag{6}$$

where $d_{ij}(x,y,t)$ can be interpreted as weights on the gradient field. Intuitively, the main idea is to identify features such as borders and exploit local coherence to guide the integration through smooth and coherent areas, avoiding large discontinuities and noisy points. To this end, the affine transformation D is obtained considering g_x and g_y and calculating (for each (x,y,t)) the 2×2 matrix

$$H(x,y,t)=(g_x\ g_y)^T(g_x\ g_y). \tag{7}$$

The resulting matrix H has an orthonormal basis of eigenvectors v_1 and v_2, where v_1 is parallel to the local gradient and v_2 is perpendicular to it. H can be rewritten as

$$H=[v_1\ \ v_2]\begin{bmatrix}\mu_1 & 0\\ 0 & \mu_2\end{bmatrix}\begin{bmatrix}v_1^T\\ v_2^T\end{bmatrix}. \tag{8}$$

In order to penalize large gradients, H eigenvectors are modified following [17],

$$\begin{cases}\lambda_2=1\\ \lambda_1=\begin{cases}1 & \text{if } \mu_1=0\\ \beta+1-\exp(-3.315/\mu_1^4) & \text{if } \mu_1>0\end{cases}\end{cases} \tag{9}$$

and finally the tensor is obtained preserving H orientation but with the modified eigenvalues:

$$D=[v_1\ \ v_2]\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}v_1^T\\ v_2^T\end{bmatrix}. \tag{10}$$

The parameter β in Eq. (9) was introduced in [9] to ensure positive-definiteness of D but was not mentioned in [16,18]. We set $\beta=0.02$ in the presented experiments in order to maintain consistency with [9]. In addition, we compared the results to those obtained for $\beta=0$ (as is described in [16,18]); for the set of experiments we performed, the results were equivalent.

Until now, two known integration strategies have been described. The first (LS method) is one of the simplest and computationally more efficient approaches to the problem and the second one (Anisotropic Diffusion) is a more complex and robust approach. Both were developed for two dimensional problems where the gradient field is a function of the spacial coordinates i.e. $g_x(x,y)$ and $g_y(x,y)$. Nevertheless, these approaches can be trivially generalized for the case of time dependant gradient fields by performing the integration for each time t independently and thus $z(x,y,t)\leftarrow(g_x(x,y,t), g_y(x,y,t))$ for each $t\in[0\ T]$. In this work, we will explore non-trivial generalizations where the relation of the gradient field over the time is exploited.

Particularly we will focus on two cases, the inspection of scenes that vary at constant speed and scenes that vary at constant acceleration ratios. We focus on these two quantities because they have an important physical meaning, but the presented theory is general and the consideration of higher order derivatives is straightforward.

Instead of minimizing the expressions given in Eqs. (3) and (6) for each frame independently, the idea is to consider

$$E[u]=\int_{\Omega\times[0\ T]}((1-\lambda)\left(|u_x-g_x|^2+|u_y-g_y|^2\right)+\lambda|\mathbf{d}^{\boldsymbol{\alpha}}\mathbf{u}/\mathbf{dt}^{\boldsymbol{\alpha}}|^2)\,dx\,dy\,dt \tag{11}$$

or

$$E[u]=\int_{\Omega\times[0\ T]}((1-\lambda)\left(d_{11}(u_x-g_x)^2+(d_{12}+d_{21})(u_x-g_x)(u_y-g_y)+d_{22}(u_y-g_y)^2\right)+\lambda|\mathbf{d}^{\boldsymbol{\alpha}}\mathbf{u}/\mathbf{dt}^{\boldsymbol{\alpha}}|^2)\,dx\,dy\,dt \tag{12}$$

[1] We use $\nabla_{xy}=(\partial/\partial x,\partial/\partial y)$ to highlight that the operator is considered in the Ω domain.