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Sequential contests with first and secondary prizes

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HIGHLIGHTS

- We study two-stage Tullock contests with first and secondary prizes.
- The secondary prize is awarded in the first stage according to the players' efforts in that stage.
- The first prize is awarded in the second stage according to the players' efforts in both stages.
- The players' marginal effort cost in the first stage is higher than in the second stage.
- We show that the players' expected payoffs increase in their marginal effort cost.

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ABSTRACT

We study a sequential two-stage Tullock contest with two asymmetric players. The players compete for two prizes; the player with the highest effort in the first stage wins the secondary prize while the player with the highest total effort in both stages wins the first prize. Both players have the same cost functions where the marginal cost in the first stage is higher than in the second one. We analyze the subgame perfect equilibrium of this contest and reveal a paradoxical behavior such that the players' utilities increase in their marginal effort cost.

1. Introduction

A war might have several battles in which the winner is not necessarily the winner of the war. A war with several battles is an example of a multi-stage contest in which one of the contestants wins the first (main) prize at the end of the contest but each of the other contestants including the winner of the first prize may win secondary prizes during the contest. We can find several such real-life contests with secondary prizes. A well-known example of a contest with secondary prizes is the Tour de France which is an annual multi-stage bicycle race. In this contest, the rider with the lowest aggregate time over all the stages wins the first prize (the prize for the general classification). However, the rider who wins the race containing climbs wins a secondary prize (the prize for the mountain classification) and there are also other secondary prizes (the prizes for the minor classifications). Another example is a political race or an election in which a party member competes to be the party's candidate for head of government (first prize) and also competes to be elected together with his party as part of the government

In this paper, we study a two-stage Tullock contest with two contestants in which a contestant who exerts a higher total effort over the two stages has a higher probability to win the first prize, while a contestant who exerts a higher effort in the first stage only, has a higher probability to win the secondary prize.[1] In order that the contestants' decision about the effort allocation over the two stages be non-trivial we assume that the contestants' marginal effort cost in the first stage is higher than in the second one.[2] Note that if the relation between the contestants' effort cost in both stage is lower in the first stage than in the second one, then both

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[1] The present paper shows only one way of allocating two asymmetric prizes in a Tullock contest. In the literature, there are several ways to allocate k prizes in such a contest (see, for example Berry, 1993; Clark and Riis, 1996, 1998).

[2] There is much evidence that, as in our model, contestants strategically allocate their resources in multi-stage contests (see, for example Harbaugh and Klumpp, 2005; Amegashie et al., 2007; Sela and Erez, 2013).

contestants will allocate efforts in the first stage only since an effort in the first stage yields winning the first and secondary prizes while an effort in the second stage yields winning the first prize only.[3]

We assume that the contestants are asymmetric, namely, they have different values of winning for different secondary prizes, but they have the same value for the first prize. In that case, we have three forms of a subgame perfect equilibrium: 1. Both contestants are active in both stages, 2. both contestants are active in the first stage, 3. one contestant is active in both stages and the other is active in the first one only. We study only the two cases in which both contestants are active in both stages or both contestants are active in the first one, since the analysis of these cases is sufficient for arriving at conclusions. We obtain that the contestants' expected payoffs are not the same when they are both active in both stages as when they are both active in the first stage only. We compare these expected payoffs and find a paradoxical result according to which both contestants' expected payoffs are higher when they are active in both stages than when they are active in the first stage only although their marginal effort costs are higher when they both are active in both stages. In other words, both contestants' expected payoffs are higher when they have higher (marginal) effort costs.

2. The model

Consider a two-stage Tullock contest with two contestants, 1 and 2 (see Tullock, 1980). In the first stage contestant i's value for the secondary prize is v_i. If both contestants exert efforts x_1, x_2 in the first stage, then the contestants' probabilities of winning are $p_1 = \frac{(x_1)^r}{(x_1)^r+(x_2)^r}, 0 < r < 2$ and $p_2 = 1 - p_1$. The cost of effort x_i for contestant i in the first stage is $c(x_i) = \beta x_i, \beta > 1$. Contestants 1 and 2 compete against each other also in the second stage where both contestants have the same value w, $w > v_i, i = 1, 2$ for the first prize. If both contestants exert efforts of y_1, y_2 in the second stage then the contestants' probabilities of winning are $p_1 = \frac{(x_1+y_1)^r}{(x_1+y_1)^r+(x_2+y_2)^r}$ and $p_2 = 1 - p_1$ where x_1, x_2 are the contestants' efforts in the first stage. The cost of effort y_i for contestant i in the second stage is normalized to be $c(y_i) = y_i$.

2.1. Case A: the contestants are active in both stages

2.1.1. The second stage

The maximization problem of contestant 1 in the second stage is

$$\max_{y_1} w \frac{(x_1+y_1)^r}{(x_1+y_1)^r+(x_2+y_2)^r} - y_1$$

Similarly, the maximization problem of contestant 2 is

$$\max_{y_1} w \frac{(x_2+y_2)^r}{(x_1+y_1)^r+(x_2+y_2)^r} - y_2$$

The F.O.C. are[4]

$$w \frac{r(x_1+y_1)^{r-1}(x_2+y_2)^r}{((x_1+y_1)^r+(x_2+y_2)^r)^2} = 1 \quad (1)$$

$$w \frac{r(x_1+y_1)^r(x_2+y_2)^{r-1}}{((x_1+y_1)^r+(x_2+y_2)^r)^2} = 1$$

By symmetry of the contestants in the second stage, we have, $x_1 + y_1 = x_2 + y_2$, and then from (1) we obtain

$$\frac{wr}{4(x_2+y_2)} = 1$$

Thus, the contestants' equilibrium strategies in the second stage are

$$y_i = \frac{wr}{4} - x_i, i = 1, 2$$

The necessary conditions that both contestants exert efforts in the second stage are $x_i < \frac{wr}{4}, i = 1, 2$. Then, we obtain that

$$x_i + y_i = \frac{wr}{4}$$

The contestants' expected payoffs in the second stage are

$$u_i(x_i) = \frac{w(2-r)}{4} + x_i, i = 1, 2$$

2.1.2. The first stage

The maximization problem of contestant 1 in the first stage is

$$\max_{x_1} v_1 \frac{(x_1)^r}{(x_1)^r+(x_2)^r} + u_1(x_1) - \beta x_1$$
$$= \max_{x_1} v_1 \frac{(x_1)^r}{(x_1)^r+(x_2)^r} + \frac{w(2-r)}{4} + x_1 - \beta x_1$$

and the maximization problem of contestant 2 is

$$\max_{x_2} v_2 \frac{(x_2)^r}{(x_1)^r+(x_2)^r} + \frac{w(2-r)}{4} + x_2 - \beta x_2$$

The F.O.C. are

$$v_1 \frac{r(x_1)^{r-1}(x_2)^r}{((x_1)^r+(x_2)^r)^2} = \beta - 1$$
$$v_2 \frac{r(x_1)^r(x_2)^{r-1}}{((x_1)^r+(x_2)^r)^2} = \beta - 1$$

Thus, the contestants' equilibrium strategies in the first stage are

$$x_1 = \frac{(v_1)^{r+1}(v_2)^r}{(\beta-1)((v_1)^r+(v_2)^r)^2}$$
$$x_2 = \frac{(v_1)^r(v_2)^{r+1}}{(\beta-1)((v_1)^r+(v_2)^r)^2}$$

The necessary and sufficient conditions that $y_i > 0, i =$ 1,2 are

$$x_1 = \frac{(v_1)^{r+1}(v_2)^r}{(\beta-1)((v_1)^r+(v_2)^r)^2} < \frac{wr}{4}$$
$$x_2 = \frac{(v_1)^r(v_2)^{r+1}}{(\beta-1)((v_1)^r+(v_2)^r)^2} < \frac{wr}{4}$$

Thus, the subgame perfect equilibrium in which both contestants exert efforts in both stages exists if

$$\beta > \max\{1 + \frac{4(v_1)^{r+1}(v_2)^r}{wr((v_1)^r+(v_2)^r)^2}, 1 + \frac{4(v_1)^r(v_2)^{r+1}}{wr((v_1)^r+(v_2)^r)^2}\} \quad (2)$$

The contestants' expected payoffs are then

$$\pi_1^A = \frac{(v_1)^{2r+1} + (1-r)(v_1)^{r+1}(v_2)^r}{((v_1)^r+(v_2)^r)^2} + \frac{w(2-r)}{4} \quad (3)$$
$$\pi_2^A = \frac{(v_2)^{2r+1} + (1-r)(v_2)^{r+1}(v_1)^r}{((v_1)^r+(v_2)^r)^2} + \frac{w(2-r)}{4}$$

Proposition 1. *In the asymmetric two-stage Tullock contest if the ratio of the contestants' marginal effort costs in the first and second stages is larger than or equal to* $\max\{1 + \frac{4(v_1)^{r+1}(v_2)^r}{wr((v_1)^r+(v_2)^r)^2}, 1 + \frac{4(v_1)^r(v_2)^{r+1}}{wr((v_1)^r+(v_2)^r)^2}\}$, *there is a subgame perfect equilibrium in which both contestants are active in both stages.*

[3] In our two-stage contest there is a synergy between the stages since the effort of the first stage affects winning the prize awarded in the first stage as well as the one awarded in the second stage. The literature suggests other reasons for the occurrence of synergy in multi-stage contests (see, for example Kovenock and Roberson, 2009; Ryvkin, 2011; Sela, 2017).

[4] For all the maximization problems, the S.O.C. is satisfied for the same values of r as in the standard Tullock contest.