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Phase unwrapping by accumulation of residual maps

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ABSTRACT

We present a path independent (global) algorithm for phase unwrapping based on the minimisation of a robust cost function. The algorithm incorporates an outlier rejection mechanism making it robust to large inconsistencies and discontinuities. The proposal consists on an iterative incremental scheme that unwraps a sub-estimation of the residual phase at each iteration. The sub-estimation degree is controlled by an algorithm's parameter. We present an efficiently computational multigrid implementation based on a nested strategy: the process is iterated by using multiple resolutions. The proposal's performance is demonstrated by experiments with synthetic and real data, and successfully compared with algorithms of the state of the art.

1. Introduction

Fringe analysis consists on the computation of the phase map from an image, or image set, with a sinusoidal fringe pattern. Such a phase is associated with a physical quantity to measure; for example, the three-dimensional topography of an optical surface. Phase unwrapping is an important process in fringe analysis and therefore an active research topic. Energy minimisation based approaches are among the most robust and efficiently computational techniques for this purpose [1–5]. Algorithms based on the minimisation of robust potentials have shown to be capable of unwrapping discontinuous phase maps [6,7]. Despite this capability, there are two major drawbacks of the energy minimisation based algorithms: their computational efficiency depends on the problem size and the differentiable energies lead one to algorithms with a slow reduction of the low-frequency error's components. In this work we propose a new algorithm for phase unwrapping constructed on the robust method proposed in [6]. Our method preserved the advantages of the original formulation, improves its slow convergence and avoids a reduction of the dynamic range of the solution.

Before to presents a brief review of the method in [6], we introduce our notation and important definitions. Phase unwrapping is an inverse problem which direct one (wrapping) can be stated as follows: let ϕ be a phase map defined in a regular lattice $\mathcal{L}$ and let g be its wrapped version such that

$$g_r = W\phi_r; \tag{1}$$

where r indicates a pixel position in $\mathcal{L}$ and

$$Wz \stackrel{\text{def}}{=} z + 2n\pi \tag{2}$$

is the non-linear wrapping operator, with n an integer such that $Wz \in (-\pi, \pi]$. The W operator has the following properties:

$$\begin{aligned} f &= W^{-1}Wf, \\ Wf &= WWf, \\ W^{-1}f &= W^{-1}W^{-1}f, \\ \sin f &= \sin(Wf); \end{aligned} \tag{3}$$

where W^{-1} denotes the unwrapping operator. We note that in general $f \neq WW^{-1}f$.

The aim of this work is to propose a procedure for implementing the unwrapping operator. Let

$$\mathcal{N}_r = \{s \in \mathcal{L} : \|r - s\|_2 = 1\} \tag{4}$$

be the set of first neighbour pixels to the pixel r. Hence, let r and $s \in \mathcal{N}_r$ be two neighbour pixels in the phase map ϕ, if

$$|\phi_r - \phi_s| < \pi, \tag{5}$$

then the first differences of the true phase can be computed by wrapping the first differences of the wrapped phase [8]:

$$\phi_r - \phi_s = W(g_r - g_s). \tag{6}$$

Since noise and phase jumps may introduce large phase differences such that $|\phi_r - \phi_s| > \pi$, then in [6] is proposed a robust phase unwrapping method based on the minimisation of

$$U(f, \omega; \rho) = \frac{1}{2}\sum_{r \in \mathcal{L}}\sum_{s \in \mathcal{N}_r}\left\{\omega_{rs}^2[(f_r - f_s - \rho_{rs})^2 + \lambda(f_r - f_s)^2] + \mu(1 - \omega_{rs})^2\right\}, \tag{7}$$

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where λ and μ are positive parameters of the algorithm, and we define

$$\rho_{rs} \stackrel{\text{def}}{=} W(g_r - g_s). \tag{8}$$

The data term (first one) in (7) is a weighted version of the regularised least square potential [1,2]. The second term [membrane potential: $(f_r - f_s)^2$] penalises large local variations on the unwrapped phase [3,4]. In the minimisation of (7), the weights $\omega_{rs} \in [0,1]$ are computed by a balance between the data and the membrane terms *versus* the value of the μ parameter; *i.e.*, if the magnitude of, either, the residual $f_r - f_s - \rho_{rs}$ or the phase difference $f_r - f_s$ is too large,

$$(f_r - f_s - \rho_{rs})^2 + \lambda(f_r - f_s)^2 > \mu, \tag{9}$$

then, for reducing the cost function value, the data and membrane terms are discarded by setting $\omega_{rs} = 0$. One can note that the membrane term introduces a smoothing effect on the final unwrapped phase product of the first differences penalisation. However, as was pointed in [3], the use of the membrane potential for noise reduction purposes also reduces the dynamical range of the solution. This effect can be noted if we try to unwrap the phase in Fig. 1(a). Fig. 2 shows the computed unwrapped phase (first row) and the rewrapped phase (second row) by minimising the energy (7) with a Gauss–Seidel scheme after 25,000 and 500,000 iterations; columns (a) and (b), respectively. It is evident the computational inefficiency of the Gauss–Seidel solver for reducing low frequency errors (residuals).

In this work we consider that denoising can be achieved in a pre-processing stage (using a narrowband quadrature filter-bank) or in a post-processing stage (filtering the unwrapped phase). This we focus on the development of an efficiently phase unwrapping method that preserves the dynamic range of the solution. Our proposal is based on a the robust energy (7) and consists on an incremental scheme where we compute an unwrapped approximation of the current residual at each iteration. The computational performance is improved by using an efficient multigrid strategy.

2. Phase unwrapping method by accumulation of residual maps (ARM)

Initially, we assume that an initial unwrapped phase $f^{(0)}$ is available, then our aim is to estimate an unwrapped residual phase δ such that

$$\phi_r = f_r^{(0)} + \delta_r. \tag{10}$$

Hence, if (5) holds, we have from (6) and (10):

$$\delta_r - \delta_s = \tilde{\rho}_{rs}; \tag{11}$$

where we set

$$\tilde{\rho}_{rs} = \rho_{rs} - (f_r^{(0)} - f_s^{(0)}) \tag{12}$$

with ρ_{rs} defined in (8). Since noise and phase jumps may introduce large phase differences such that $\exists(r,s) : |\phi_r - \phi_s| > \pi$, then (6) can be contravened by some first neighbour pixel pairs. Therefore, we propose to compute the residual phase by minimising the robust cost function for the residual phase map:

$$\{\delta, \omega\} = \underset{\delta,\omega}{\operatorname{argmin}}\, U(\delta, \omega; \tilde{\rho}). \tag{13}$$

The solution to (13) can be computed by alternating minimisations with respect to δ and ω [6,9,12]. Thus, if ω is fixed; the solution of the positive-definite diagonal-dominant linear system that results of $\partial U/\partial\delta_r = 0$ can be computed with a Gauss–Seidel iterative scheme:

$$\delta_r = \frac{\sum_{s \in \mathcal{N}_r} \omega_{rs}^2[(\delta_s + \tilde{\rho}_{rs}) + \lambda\delta_s]}{\sum_{s \in \mathcal{N}_r} \omega_{rs}^2[1+\lambda]}. \tag{14}$$

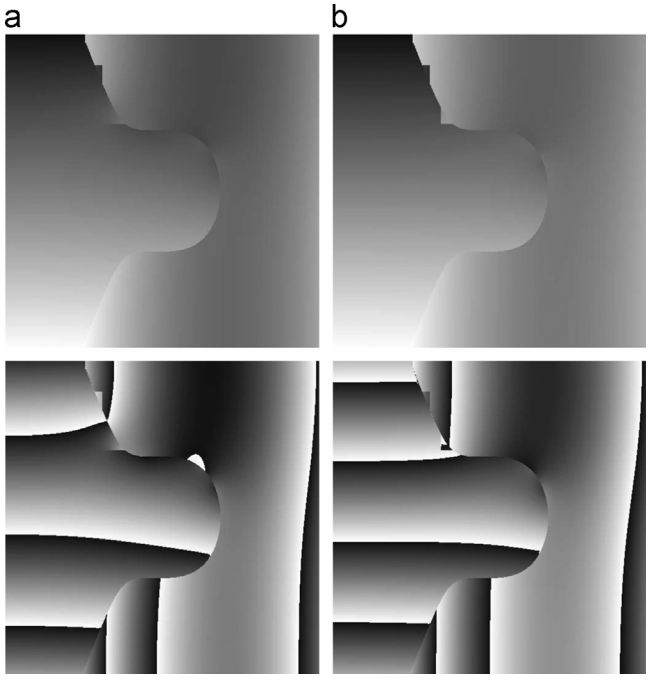

Fig. 2. Unwrapped phases (first row) and rewrapped phases (second row) computed by minimising (7) with a Gauss–Seidel scheme after: (a) 25,000 iterations, computational time = 127.2 s and MSE = 0.669. (b) 500,000 iterations, computational time = 2481.1 s and MSE = 0.241.

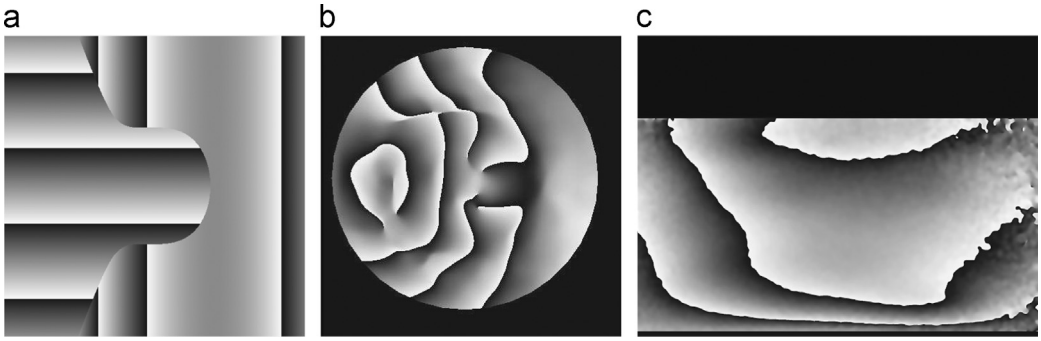

Fig. 1. Test wrapped phases. (a) Phase1 (synthetic), 512 × 512 pixels. (b) Phase2 (cornea), 1920 × 1440 pixels. (c) Phase3 (iron plate), 423 × 423 pixels.