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A model of sales with differentiated and homogeneous goods

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HIGHLIGHTS

- Product differentiation has substantive consequences for price dispersion.
- The bounds on prices depend on the equilibrium expected price.
- Expected profits and the bounds on prices decrease with the mass of bargain seekers.
- The setting nests the classical model of sales as a special case.

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ABSTRACT

This paper extends the classical model of sales (Varian, 1980; Rosenthal, 1980) by adding product differentiation. Instead of uninformed (i.e. loyal) customers, our setting features "variety seekers". These consumers regard the products as imperfect substitutes. As in the original model, the firms also serve "bargain seekers" who buy the cheapest product. The discontinuous demand structure precludes any pure-strategy equilibria. We characterize the symmetric mixed-strategy equilibrium of the modified game. In contrast to the original model, the upper bound on prices and the equilibrium expected profits are decreasing in the mass of bargain seekers.

1. Introduction

This paper studies how horizontal product differentiation affects price dispersion in oligopolistic industries. We modify the classical model of sales (Varian, 1980; Rosenthal, 1980) by replacing loyal customers with "variety seekers". From the perspective of these consumers, products are imperfect substitutes. As in the original setting, firms also serve "bargain seekers" who always buy the cheapest good. The resulting demand discontinuity precludes any pure-strategy Nash equilibria. However, it may induce firms to randomize their prices. We characterize this mixed-strategy equilibrium and investigate its properties.

Product differentiation has substantive consequences for the implied distribution of prices. In the classical model of sales, the presence of loyal consumers generates simple expressions for the upper bound of the support and the expected profits that are independent of the size of the market for perfect substitutes. Our modification invalidates these results. We show that if firms compete in both markets, the upper bound on prices and the expected profits will decrease with the mass of bargain seekers.

Our paper can also be viewed as a generalization of the classical model of sales. When variety seekers regard the products as unsubstitutable, they provide each firm with a captive market. In this special case, our game reduces to the setting of Rosenthal (1980), and yields identical conclusions.

The purpose of the classical model of sales was to explain observed dispersion of market prices. It has been applied to fields such as trade (Baye and de Vries, 1992), banking (Broecker, 1990), marketing (Raju et al., 1990) and finance (Dennert, 1993). The full equilibrium set of this game was characterized by Baye et al. (1992). The original setting has been extended in several directions. Examples of such extensions include the introduction of advertising (Chioveanu, 2005; Arnold and Zhang, 2014) and consumer search (Stahl, 1988, 1996; Chen and Zhang, 2011).

While a number of papers have examined product differentiation in oligopolistic markets (Caplin and Nalebuff, 1991; D'Aspremont et al., 1979; Bester, 1992), they typically focus on pure-strategy equilibria. There is little research on the implications for price dispersion. The paper that comes closest to ours is Sinitsyn (2008). Sinitsyn's model features perfectly loyal consumers and a market for differentiated goods, but lacks bargain seekers. As a result, any mixed-strategy equilibria that may arise in his setting have a discrete support.

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2. Setting

Consider two firms, A and B. They produce outputs q_A and q_B at zero cost, and compete by simultaneously choosing their prices, p_A and p_B. The products are sold to two types of consumers: variety seekers and bargain seekers. Both types maximize quasi-linear utility functions.

- **Variety seekers**. These consumers are assumed to have quadratic subutility:

  $$u^I(q_A^I, q_B^I) = \alpha(q_A^I + q_B^I) - \frac{\beta}{2}(q_A^I)^2 - \frac{\beta}{2}(q_B^I)^2 - \gamma q_A^I q_B^I,$$

  where $\alpha > 0$ and $\beta > \gamma \geqslant 0$. Their mass is normalized to one. It is easy to show that j's demand from the variety seekers is

  $$q_j^I = \begin{cases} \psi - \mu p_j + \eta p_{-j} \text{ if } p_j \leqslant (\psi + \eta p_{-j})/\mu \\ 0 \text{ if } p_j > (\psi + \eta p_{-j})/\mu, \end{cases}$$

  where $\psi = \alpha/(\beta+\gamma)$, $\mu = \beta/(\beta^2-\gamma^2)$ and $\eta = \gamma/(\beta^2-\gamma^2)$.
- **Bargain seekers**. These consumers are analogous to the switchers in the classical model of sales. Their subutility is

  $$u^{II}(q_A^{II}, q_B^{II}) = \rho(q_A^{II} + q_B^{II}),$$

  where $\rho(\cdot)$ is an increasing function. Suppose that each buys at most one unit of the commodity, and let their mass be m. Provided that the bargain seekers participate, they will always purchase the cheapest good. When $p_A = p_B$, we assume that each firm sells to $m/2$ of them. Hence, firm j's demand from the bargain seekers is

  $$q_j^{II} = \begin{cases} m \text{ if } p_j < p_{-j} \\ 0 \text{ if } p_j > p_{-j} \\ m/2 \text{ if } p_j = p_{-j}. \end{cases}$$

The above demand structure implies that j's profit is given by

$$\pi_j(p_j, p_{-j}) = \begin{cases} \left(\psi - \mu p_j + \eta p_{-j}\right) p_j \text{ if } p_j \in \left(p_{-j}, (\psi + \eta p_{-j})/\mu\right] \\ \left(\psi - \mu p_j + \eta p_{-j} + m\right) p_j \text{ if } p_j \in \left[0, p_{-j}\right) \\ (\psi - \mu p + \eta p + m)\, p/2 \text{ if } p_j = p_{-j} = p. \end{cases}$$

2.1. Non-existence of pure-strategy equilibria

In this section, we argue that our game has no equilibria in pure strategies. Symmetry allows us to focus on an arbitrary firm $j \in \{A, B\}$.

First, we introduce notation for what the best responses and the equilibrium prices would be if firms faced the two types of consumers separately. Imagine that there were only variety seekers. It is trivial to show that, given p_{-j}, firm j's profit-maximizing price would be

$$BR_j^I(p_{-j}) = \frac{\psi + \eta p_{-j}}{2\mu}, \quad p_{-j} \geqslant 0.$$

These best responses would induce a Nash equilibrium in which each firm charges a price $p^I = \psi/(2\mu - \eta)$. If, on the other hand, all consumers were bargain seekers, j's best response would be to marginally undercut the opponent's price:

$$BR_j^{II}(p_{-j}) = p_{-j} - \varepsilon, \quad p_{-j} > 0.$$

In the Nash equilibrium of that game, each firm would charge $p^{II} = 0$.

Now we establish the non-existence of pure-strategy Nash equilibria in a setting with both types of consumers. To achieve this, we find a profitable deviation p_j' for each price pair (p_j, p_{-j}).

- $p_j \geqslant p_{-j} > p^I$. Suppose that j charges $p_j' = BR_j^I(p_{-j}) < p_j$ instead. This price would maximize j's revenue from the variety seekers. Moreover, as $p_{-j} > p^I$, we have $BR_j^I(p_{-j}) < p_{-j}$. Thus, p_j' would also enable j to start selling to the bargain seekers.
- $p_j \geqslant p^I > p_{-j}$. Again, firm j can increase its profit by setting $p_j' = BR_j^I(p_{-j})$. Note that $p^I > p_j' > p_{-j}$, so all bargain seekers would continue buying from $-j$. However, p_j' would maximize j's revenue from the variety seekers.
- $p^I \geqslant p_{-j} > p_j \geqslant 0$. Now firm j can improve its payoff by charging $p_j' = BR_j^{II}(p_{-j}) = p_{-j} - \varepsilon$. This price would increase j's revenue from the bargain seekers (since they will keep buying from j, but at a higher price), as well as its revenue from the variety seekers (since this revenue is concave in j's price and $BR_j^I(p_{-j}) > p_{-j} > p_j$).
- $p^I \geqslant p_j = p_{-j} > 0$. In this case, j's profitable deviation is to charge $p_j' = BR_j^{II}(p_{-j}) = p_{-j} - \varepsilon$. Marginally undercutting $-j$'s price would allow j to capture the entire revenue from the bargain seekers.
- $p_j = p_{-j} = 0$. Finally, when $-j$ sets its price to 0, firm j's optimal price would be $p_j' = BR_j^I(0) = \psi/(2\mu) > 0$. While p_j' would still earn j zero revenue from the bargain seekers, this price will maximize j's revenue from the variety seekers.

The above five scenarios exhaust all candidates for pure-strategy Nash equilibria. Hence, the game does not have such equilibria. This result is summarized in Proposition 1.

Proposition 1. *The modified model of sales does not have a pure-strategy Nash equilibrium.*

2.2. Mixed-strategy equilibrium

As the classical model of sales, the above game may have an equilibrium in which firms randomize their prices. We focus on the equilibrium involving symmetric mixed strategies. In the Appendix, we show that such strategies must have a continuous atomless support. We denote its lower bound by l, and its upper bound by h. The equilibrium mixing distribution is described by a continuous, strictly increasing cumulative distribution function (cdf), $F(p)$, that satisfies $F(l) = 0$ and $F(h) = 1$. Let the corresponding probability density function (pdf) be $f(x)$.

2.2.1. Cumulative distribution function

We pin down the functional form of the cdf from the expression for j's expected payoff. Suppose that $-j$ randomizes according to $F(\cdot)$, and let $\bar{p}_{-j}$ be its expected price:

$$\bar{p}_{-j} = \int_l^h p_{-j} f(p_{-j}) dp_{-j}. \tag{1}$$

When firm j charges a price p_j, its expected payoff will be

$$\bar{\pi}_j(p_j) = \left(\psi - \mu p_j + \eta \bar{p}_{-j}\right) p_j + \left[1 - F(p_j)\right] m p_j. \tag{2}$$

The first term is the expected revenue from the variety seekers. Note that it is maximized when $p_j = BR_j^I(\bar{p}_{-j})$. The second term is the expected revenue from the bargain seekers.

Firm j is willing to use a mixed strategy only if its expected profit is independent of p_j. Imposing $\bar{\pi}_j(p_j) \equiv \bar{\pi}_j$ on (2) and solving for $F(p_j)$, we get

$$F(p_j) = 1 - \frac{\bar{\pi}_j}{m p_j} + \frac{\psi - \mu p_j + \eta \bar{p}_{-j}}{m}. \tag{3}$$

To complete the characterization of the mixed-strategy equilibrium, we need to determine $\bar{p}_{-j}$, $\bar{\pi}_j$, h and l. We must also verify that (3) is strictly increasing, i.e. $f(p_j) > 0$ for $p_j \in [l, h)$.