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Portfolio choice in personal equilibrium

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HIGHLIGHTS

- We study optimal portfolio choice where reference point arises endogenously in personal equilibria.
- In addition to CPE, UPE is also linked to the rank-dependent utility (RDU) in the context of portfolio choice.
- The equivalence between UPE and RDU only applies in the characterization of the *optimal* risky choice.
- The non-uniqueness of UPE is caused by non-convexities of the choice set.

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ABSTRACT

This paper finds that in portfolio choice where reference point arises endogenously in personal equilibria, investors behave as if they had a concave probability weighting function. This finding establishes a link between the reference-dependent utility and the rank-dependent utility theories.

1. Introduction

In a stimulating paper, Kőszegi and Rabin (2007) (KR, henceforth) explored how individuals with expectation-based reference-dependent preferences make a risky choice. In their model, individuals care about consumption utility as well as gain–loss utility (i.e., utility over deviations from the reference), and the reference is the full distribution of the payoff reflecting individuals' expectations. KR provide a solution framework for the formation of expectations-based reference, in which the individual knows exactly how he or she will behave in any future contingency and his or her reference point reflects this actual behavior. KR's framework has inspired numerous applications. Among others, Heidhues and Kőszegi (2008) use this framework to study the Salop price competition; Herweg et al. (2010) apply it to re-design the employee compensation contracts; Karle and Peitz (2014) employ it to study the firm competition with asymmetric information regarding consumer tastes. In financial markets, Pagel (2016) explores the asset-pricing implications of KR's solution in a Lucas-tree model with dynamic asset allocations, while Pagel (2018) uses KR's framework to solve a life-cycle portfolio choice problem in which the investor experiences loss-averse utility over news.

Despite the existing applications, the implications of KR's framework on optimal portfolio choice are not fully investigated. Pagel (2016, 2018) use KR's framework in intermediate steps to solve the portfolio choice problems. However, she obtained the unique solution only for power and log utility functions under lognormal distributions for risky assets. It is not clear whether the solution would be unique for all concave utility functions and all continuous distributions, and how to characterize the optimal portfolio weights in KR's framework in the more general setting. The purpose of our paper is to address these questions.

Specifically, we offer an explicit characterization of the solution to the portfolio choice problem in KR's framework under

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a general setting. KR introduces two specific solution concepts. One is the "unacclimating personal equilibrium" (UPE, henceforth), defined for the case where the choice is made based upon the reference, and equilibrium is achieved when the optimal choice coincides exactly with the reference. The other one is the "choice-acclimating personal equilibrium" (CPE, henceforth), defined for the case where the individual first sets the choice as the reference, and then optimizes over the choices. As KR argued, UPE arises in a context where individuals expect a choice only if they are willing to follow it through, while CPE applies in a context where individuals commit to a choice before outcomes occur. We show that investors in both UPE and CPE behave as if they had a concave probability weighting function, as axiomatized by Yaari (1987). This characterization is interesting because it establishes an equivalence between nonstandard utility under correct beliefs and standard utility under distorted beliefs.

The equivalent concave probability weighting function implies that the choice in UPE is unique. Only with uniqueness, we are able to determine the UPE by virtue of the first-order condition and further translate this condition into a rank-dependent structure. This result is in contrast to the previous finding of multiple UPE choices under a discrete choice set, as shown in KR.[1] The change in the property of UPE is driven by the change in the structure of the choice set: when the choice set contains only discrete strategies, the adjustment of reference in the UPE is likely to get stuck on some choice that is not globally optimal, yielding multiple UPE. In contrast, when the choice set contains the continuum of all possible strategies (as in the context of portfolio choice), the sub-optimal equilibria are easily disturbed, and the UPE converges to a unique strategy. This result thus enriches our understanding on the implications of UPE.

This paper is not the first attempt to connect KR's reference-dependent utility theory to Yarri's dual theory, or more broadly, Quiggin (1982)'s rank-dependent utility theory. Masatlioglu and Raymond (2016) focus on CPE and show in their Proposition 4 (p. 2767) that for any risky choice, if investors set the distribution of the choice as the reference, then their evaluation of the choice with the reference is equivalent to an evaluation with a concave probability weighting function.[2] One novelty of our paper is that we also study UPE, for which the equivalent probability weighting function is more difficult to observe because it arises only for the optimal choice. Moreover, we show that the equivalent probability weighting function for UPE is less concave than that for CPE, which is consistent with the prediction in Proposition 8 of KR that investors in UPE are less risk averse than in CPE.

2. The concept of UPE and CPE

We model investors' reference-dependent utility in the manner of KR. Let the investor's risky wealth be $\tilde{w}$ and her reference be $\tilde{r}$. For any realized outcome $\tilde{w} = w$, the investor gets an intrinsic consumption utility $u(w)$, and a gain–loss utility

$$E[\mathscr{R}(u(w) - u(\tilde{r}))|\tilde{w} = w]. \tag{1}$$

The gain–loss utility describes the feeling of the investor when she compares the wealth outcome with the reference $\tilde{r}$. Denote the cumulative distribution functions (cdfs) of $\tilde{w}$ and $\tilde{r}$ by F and G respectively. In KR, $\tilde{w}$ and $\tilde{r}$ are assumed to be independent, and the gain–loss utility is calculated by comparing an outcome w to every possible outcome of $\tilde{r}$.[3] The investor's expected reference-dependent utility is given by

$$E[v(\tilde{w};\tilde{r})] = \int\int u(w) + \mathscr{R}(u(w) - u(r))dF(w)G(r), \tag{2}$$

where u is a concave von Neumann–Morgenstern utility function and $\mathscr{R}$ is a universal gain–loss value function. In a portfolio problem, the investor's risky wealth is

$$\tilde{w}(\alpha) = w_0 + \alpha\tilde{x}, \tag{3}$$

where w_0 is her initial wealth, $\tilde{x}$ is the net return of the risky asset, and α (≥ 0) is the investor's risky allocation.

In the rest of the paper, UPE is defined for the case where the stochastic outcome generated by utility maximization conditional on a reference coincides with the reference. CPE is defined for the case where a decision is committed to before outcomes realize, and hence determines both the reference and the outcome distributions.

Definition 1. For a reference-dependent utility maximizer who needs to select the optimal risky investment, we say her choice α^U achieves a UPE, if and only if

$$\alpha^U = \arg\max_{\{\alpha\geq 0\}} E\left[v\left(\tilde{w}(\alpha);\, \tilde{w}\left(\alpha^U\right)\right)\right].$$

We say her choice α^C achieves a CPE, if and only if

$$\alpha^C = \arg\max_{\{\alpha\geq 0\}} E\left[v\left(\tilde{w}(\alpha);\, \tilde{w}(\alpha)\right)\right].$$

3. Portfolio choice in UPE and CPE

To gain tractability, we follow KR and Masatlioglu and Raymond (2016) to assume a linear gain–loss function: $\mathscr{R}(x) = \eta x$ for $x \geq 0$ and $\mathscr{R}(x) = \lambda\eta x$ for $x < 0$, where $\eta > 0$ and $\lambda > 1$. Under this assumption, an analytically amenable expression of (2) is available[4] :

$$\begin{aligned}
E[v(\tilde{w};\tilde{r})] &= E[u(\tilde{w}) + \eta(u(\tilde{w}\vee\tilde{r}) - u(\tilde{r})) + \eta\lambda(u(\tilde{w}\wedge\tilde{r}) - u(\tilde{r}))] \\
&= \int u(s)dP(\tilde{w}\leq s) + \eta\int u(s)dP(\tilde{w}\vee\tilde{r}\leq s) \\
&\quad + \eta\lambda\int u(s)dP(\tilde{w}\wedge\tilde{r}\leq s) \\
&\quad - \eta(1+\lambda)\int u(s)dP(\tilde{r}\leq s) \\
&= \int u(s)dF(s) + \eta\int u(s)d[F(s)G(s)] \\
&\quad + \eta\lambda\int u(s)d[F(s) + G(s) - F(s)G(s)] \\
&\quad - \eta(1+\lambda)\int u(s)dG(s) \\
&= \int u(s)d[F(s)(1+\eta\lambda - \eta(\lambda-1)G(s))] \\
&\quad - \eta\int u(s)dG(s).
\end{aligned} \tag{4}$$

Especially, when $\tilde{r} =^d \tilde{w}$, $F(s) = G(s)$ and (4) turns out to be

$$E[v(\tilde{w};\tilde{w})] = \int u(w)d\varphi(F(w)), \tag{5}$$

[1] KR (p. 1056) recognized that "There can be multiple UPE in a given situation – there can be multiple self-fulfilling expectations – and generically different UPE yield different expected utilities".

[2] In their Proposition 4, Masatlioglu and Raymond (2016) translate the CPE into a convex distortion of decumulative distribution function. In our paper, we translate CPE into a concave distortion of cumulative distribution function. These two kinds of distortions are equivalent.

[3] The cross-state comparison basically builds on disappointment theory. De Giorgi and Post (2011) study the case where outcomes and stochastic reference are compared state by state.

[4] We use the notation $a\vee b = \max\{a, b\}$ and $a\wedge b = \min\{a, b\}$.