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Bargaining set with endogenous leaders: A convergence result

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HIGHLIGHTS

- A new bargaining set for finite economies is introduced.
- This concept is relevant in replicated games or economies
- Differences with related concepts of bargaining sets are pointed out.
- This bargaining set converges to the set of Walrasian allocations when the economy is replicated.

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ABSTRACT

We provide a notion of bargaining set for finite economies where the proponents of objections (leaders) are endogenous. We show its convergence to the set of Walrasian allocations when the economy is replicated.

1. Introduction

The core of an economy is defined as the set of allocations which cannot be blocked or objected by any coalition. Thus, the veto mechanism that defines the core does not take into account that other agents in the economy may react to an objection and propose an alternative or counterobjection. Such two-step conception of the veto mechanism was considered by Aumann and Maschler (1964), who introduced the concept of bargaining set of a cooperative game.[1] In the definitions by Aumann and Maschler (1964) and Davis and Maschler (1963), the original objection is proposed by a "leader" that must be excluded from any counterobjecting coalition.

Geanakoplos (1978) considered sequences of transferable utility (TU) exchange economies with smooth preferences and modified the definition by Aumann–Davis–Maschler so that the "leader" was a group of agents containing a fixed (but small) fraction of the number of agents in the economy. Thus, as the number of agents grew along the sequence of economies, the number of individuals in the "leader" grew proportionately. Using nonstandard analysis, Geanakoplos showed that his bargaining set becomes asymptotically competitive as the number of agents grows. Shapley and Shubik (1984) showed that Aumann–Davis–Maschler's bargaining set is approximately competitive in replica sequences of TU exchange economies with smooth preferences. Anderson (1998) extended both Geanakoplos' result to nontransferable utility (NTU) exchange economies without smooth preferences and the Shapley and Shubik's result to non-replica sequences of NTU exchange economies with smooth preferences.

Mas-Colell (1989) considered (NTU) economies with a continuum of agents and proposed a modification of Aumann and

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[1] Maschler (1976) discussed the advantages that the bargaining set has over the core.

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Maschler's bargaining set that does not involve the concept of a leader. Dutta et al. (1989) defined the consistent bargaining set arguing that the same requirement stated for objections should also be stated for counterobjections. Later on, Zhou (1994) defined a bargaining set by imposing restrictions on the coalition that counterobjects. Under conditions of generality similar to Aumann's (1964) core equivalence theorem, Mas-Colell (1989) showed that his bargaining set characterizes the set of competitive allocations. Since both consistent and Zhou's bargaining sets are subsets of Mas-Colell's, they also equate the set of competitive allocations.

In contrast to Debreu and Scarf's (1963) core-convergence result,[2] Anderson et al. (1997) showed that Zhou's (1994) bargaining set, and consequently Mas-Colell's (1989), not necessarily converge in replica sequences of economies, no matter how nice the preferences may be. However, it is worth noting that Anderson et al.'s counterexample does not show nonconvergence of the consistent Mas-Colell bargaining set (see Anderson, 1998; p. 4).

In this paper, we provide a new definition of bargaining set. Our approach refers to scenarios where individuals are representatives of an institution, a trade union or an organization. Although our notion requires the presence of a "leader" in the objection process, it differs from the previous ones basically in two aspects. First, the leader proposing an objection has to be fully represented. This implies that no agent of the same type as the leader can participate in a counterobjection. Second, if an individual belongs to an objecting coalition, then any other agent of the same type that participates in a counterobjection is required to be better off than her homologue in the objection.

These modifications of the leader models lead us to our bargaining set convergence result. We show that an allocation is Walrasian if and only if the corresponding equal treatment allocation defined in every replicated economy cannot be blocked by a justified objection. In other words, the set of Walrasian allocations is characterized by the intersection of bargaining sets of a sequence of replicated economies.[3]

Since our bargaining set is different from those considered in the related literature (Geanakoplos, 1978; Shapley and Shubik, 1984; Anderson, 1998) neither our convergence result can be deduced from the previous ones nor vice versa.

The rest of the work is structured as follows. In Section 2, notations and preliminaries are stated. In Section 3 we present the notion of justified objections with leaders used to define our bargaining set. Section 4 contains our limit result and some concluding remarks.

2. Preliminaries, notations and some previous results

Let $\mathcal{E}$ be an exchange economy with a finite set of agents $N = \{1, \dots, n\}$, who trade a finite number m of commodities. Each consumer i has a preference relation $\succsim_i$ on the set of consumption bundles $\mathbb{R}^m_+$, with the properties of continuity, strict convexity[4] and strict monotonicity. Let $\omega_i \in \mathbb{R}^m_{++}$ denote the endowments of consumer i. So the economy is $\mathcal{E} = (\mathbb{R}^m_+, (\succsim_i, \omega_i)_{i\in N})$.

An allocation x is a consumption bundle $x_i \in \mathbb{R}^m_+$ for each agent $i \in N$. The allocation x is feasible in the economy $\mathcal{E}$ if $\sum_{i=1}^n x_i \leq \sum_{i=1}^n \omega_i$. A price system is an element of the $(m-1)$-dimensional simplex of $\mathbb{R}^m_+$. A Walrasian equilibrium for $\mathcal{E}$ is a pair (p, x), where p is a price system and x is a feasible allocation such that, for every agent i, the bundle x_i maximizes her preference relation $\succeq_i$ in the budget set $B_i(p) = \{y \in \mathbb{R}^m_+ \text{ such that } p \cdot y \leq p \cdot \omega_i\}$. We denote by $W(\mathcal{E})$ the set of Walrasian allocations for the economy $\mathcal{E}$.

A coalition is a non-empty set of consumers. An allocation y is said to be attainable or feasible for the coalition S if $\sum_{i\in S} y_i \leq \sum_{i\in S}\omega_i$. Let $x \in \mathbb{R}^{mn}_+$ be a feasible allocation in the economy $\mathcal{E}$. The coalition S blocks x if there exists an allocation y which is attainable for S, such that $y_i \succsim_i x_i$ for every $i \in S$ and $y_j \succ_j x_j$ for some member j in S. When S blocks x via y we say that (S, y) is an objection to x. A feasible allocation is efficient if it is not blocked by the grand coalition, formed by all the agents. The core of the economy $\mathcal{E}$, denoted by $C(\mathcal{E})$, is the set of feasible allocations which are not blocked or objected by any coalition of agents.

For each positive integer r, the r-fold replica economy $\mathcal{E}^r$ of $\mathcal{E}$ is a new economy with rn agents indexed by ij, with $i = 1, \dots, n$ and $j = 1, \dots, r$, such that each consumer ij has a preference relation $\succsim_{ij} = \succsim_i$ and endowments $\omega_{ij} = \omega_i$. Note that a coalition $\hat{S}$ in a replicated economy is formed by $r_i > 0$ members identical to each agent i in a non-empty subset S of $\{1, \dots, n\}$. Thus, $\hat{S} := \{ij_i \mid i \in S,\ j_i = 1, \dots, r_i\} = \bigcup_{\substack{i\in S \\ j_i\in\{1,\dots,r_i\}}} \{ij_i\}$ is actually a coalition in any replicated economy $\mathcal{E}^r$, for every $r \geq \max\{r_i, i \in S\}$.

It is known that, under the hypotheses above, the economy $\mathcal{E}$ has Walrasian equilibrium and that any Walrasian allocation belongs to the core (in particular, it is efficient). It is also known that if we repeat any Walrasian allocation when we enlarge the economy to r participants of each type, the resulting allocation is also Walrasian in the larger economy $\mathcal{E}^r$ and consequently is in the core. Moreover, as Debreu and Scarf (1963) prove, any repeated non-Walrasian allocation is objected in some replicated economy (core convergence theorem).

Addressing continuum economies, Mas-Colell (1989) provided a notion of bargaining set and showed its coincidence with the competitive allocations. This bargaining set can be straightforwardly translated to replicated economies as follows.

An objection to the allocation $(x_{ij})_{\substack{i\in N\\ 1\leq j\leq r}}$ in the economy $\mathcal{E}^r$ is defined by a coalition $\hat{S} = \{ij_i \mid i \in S,\ j_i = 1, \dots, r_i\}$, with $\max\{r_i, i \in S\} \leq r$, and a collection of consumption bundles $y = (y_{ij})_{ij\in\hat{S}}$, such that (i) $\sum_{ij\in\hat{S}} y_{ij} \leqslant \sum_{i\in S} r_i\omega_i$ and (ii) $y_{ij} \succsim_i x_{ij}$, for every $ij \in \hat{S}$ and $y_{hk} \succ_h x_{hk}$ for some $hk \in \hat{S}$. A counterobjection to $(\hat{S}, y)$ is defined by a coalition $\hat{T} = \{ij_i \mid i \in T,\ j_i = 1, \dots, a_i\}$ in $\mathcal{E}^r$ and consumption plans $(z_{ij})_{ij\in\hat{T}}$, such that (i) $\sum_{ij\in\hat{T}} z_{ij} \leqslant \sum_{i\in T} a_i\omega_i$, (ii) $z_{ij} \succ_i y_{ij}$ if consumer $ij \in \hat{T} \cap \hat{S}$ and $z_{ij} \succ_i x_{ij}$ if $ij \in \hat{T} \setminus \hat{S}$.

An objection is justified if it is not counterobjected by any coalition. $B_{MC}(\mathcal{E}^r)$ is the set of feasible allocations in $\mathcal{E}^r$ with no justified objection.

To analyze convergence properties of bargaining sets for replicated economies, in the next section we will consider that each of the n agents of the economy $\mathcal{E}$ behaves as a representative of a large enough number of identical individuals. We will also use the fact that a finite economy $\mathcal{E}$ with n consumers can be associated to a continuum economy $\mathcal{E}_c$ with n types of agents as we specify next. The set of agents in the atomless economy $\mathcal{E}_c$ is $I = [0, 1] = \bigcup_{i=1}^n I_i$, with $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$ if $i \neq n$; $I_n = \left[\frac{n-1}{n}, 1\right]$. All the agents in the subinterval I_i are of the same type i, that is, every agent $t \in I_i$ has preferences $\succsim_t = \succsim_i$ and endowments $\omega(t) = \omega_i$. In this case, $x = (x_1, \dots, x_n)$ is a Walrasian allocation in $\mathcal{E}$ if and only if the step function f_x (defined by $f_x(t) = x_i$ for every $t \in I_i$) is a competitive allocation in $\mathcal{E}_c$. Therefore, following Mas-Colell, if $x = (x_1, \dots, x_n)$ is not Walrasian, then the step function f_x does not belong to the bargaining set $B_{MC}(\mathcal{E}_c)$.

Let B^*_{MC} denote the set of equal treatment allocations in the Mas-Colell's bargaining set. The equivalence between $B_{MC}(\mathcal{E}_c)$ and

2 The core convergence is one the most commonly used tests of the price-taking assumption inherent in the definition of Walrasian equilibrium. See Anderson (1992, 2008) for surveys.

3 In a related paper, Hervés-Estévez and Moreno-Garcí (2017) obtained a convergence theorem for a bargaining set without any consideration of a leader, but under a necessary assumption of continuity of the Walrasian equilibrium correspondence instead.

4 This simplify the analysis and implies that any core allocation (and then any Walrasian allocation) is equal treatment. However, as in Debreu and Scarf (1963), our convergence result can be generalized by considering a weaker convexity that requires: if a consumption bundle z is strictly preferred to $\hat{z}$ so is the convex combination $\lambda z + (1-\lambda)\hat{z}$ for any $\lambda \in (0, 1)$.