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Finite sample performance of a long run variance estimator based on exactly (almost) unbiased autocovariance estimators

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HIGHLIGHTS

- We proposed a bias reduced long run variance estimator for testing.
- We use an almost unbiased autocovariance estimator.
- The t-statistics for the mean in a simple location model perform well.
- Simulations demonstrate promising empirical performance and suggests further theoretical analysis.

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ABSTRACT

This paper proposes a bias reduced long run variance (LRV) estimator of a univariate time series with unknown mean that addresses well known finite sample bias problems. The LRV estimator is based on the (almost) exactly unbiased autocovariance estimator proposed by Vogelsang and Yang (2016). Whereas using fixed-b critical values is known to correct downward bias in LRV estimates generated by demeaning the data, the approach we take also corrects the classic Parzen bias that is not captured by the fixed-b approach. When applied to the tests of the null hypothesis of the mean in a simple location model, a simulation study shows that the proposed LRV estimator leads to tests with less over-rejections while maintaining power at least as high and often higher as the standard robust t test based on fixed-b critical values. These simulations suggest further theoretical analysis of the bias reduction approach is warranted.

1. Introduction

The long run variance (LRV) of a covariance stationary time series is widely used in constructing test statistics for inference robust to serial correlation. Here we focus on the widely used LRV estimators based on nonparametric heteroskedasticity and autocorrelation consistent (HAC) kernel estimators as analyzed in the seminal work by Newey and West (1987) and Andrews (1991). Because kernel HAC estimators are constructed using linear combination of estimated autocovariances, well known biases in sample autocovariances contribute to biases in LRV estimators. Vogelsang and Yang (2016, VY16 hereafter) recently proposed an (almost) unbiased autocovariance estimator based on the observation that the expectation of the sample autocovariances can be written as a linear transformation of the vector of population autocovariances that involves a matrix with only deterministic entries, labeled as **A**. If the inverse linear transformation exists, an unbiased estimator of the population autocovariances is obtained by inverting the linear transformation. While VY16 find that the **A** matrix is singular, they show that an invertible transformation is obtained if at least one high lag autocovariance is neglected. This leads to autocovariance estimators that are unbiased for moving average processes and approximately unbiased for autoregressive processes. Using autocovariance estimates based on the **A**-matrix approach, we develop a bias reduced LRV estimator that we use to construct heteroskedasticity autocorrelation robust (HAR) fixed-b tests regarding the mean in a simple location model. The bias

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improvement has noticeable positive impacts on inference and leads to t-tests with higher power and less over-rejection problems at the same time. We also examine bias reduction methods in LRV estimation based on the approach in Okui (2010). We show that the approach of Okui (2010) is equivalent to scaling out the demeaning bias implied by fixed-b theory. When used in conjunction with fixed-b critical values, the Okui (2010) approach leads to tests that are equivalent to using the usual kernel LRV estimator. Note that the use of fixed-b critical values is becoming standard practice given that fixed-b critical values are known to reduce size distortions. See, among others, Kiefer et al. (2000), Kiefer and Vogelsang (2002, 2005), Sun et al. (2008), Sun and Kaplan (2011), and Müller (2014).

Using a simulation study we compare the performance of the new tests with standard tests based on kernel LRV estimators and fixed-b critical values. The new approach leads to tests with less over-rejections while maintaining power at least as high and often higher as the standard robust t-test based on fixed-b critical values. These simulations suggest further theoretical analysis of the bias reduction approach is warranted which we leave for future research.

The remainder of this paper is organized as follows. Section 2 presents the simple location model and defines the standard LRV estimator, the bias-corrected LRV estimator proposed by Okui (2010), and the **A**-Matrix LRV estimator. HAR inference is discussed in Section 3. Section 4 gives representative results from a simulation study that compares empirical null rejections and power of the standard t-statistic and its **A**-matrix version. Some technical details are given in an Appendix.

2. Model and statistics

Consider a covariance stationary univariate time series given by

$$y_t = \mu + u_t, \quad t = 1, 2, \ldots, T, \tag{1}$$

$$E(u_t) = 0,$$

$$\gamma_j = E(u_t u_{t-j}), \quad j = 0, 1, 2, \ldots$$

The LRV of u_t, which we denote by σ^2, is given by

$$\sigma^2 = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j.$$

The OLS estimator of μ is

$$\hat{\mu} = \bar{y} = T^{-1}\sum_{t=1}^{T} y_t,$$

and the OLS residuals are given by

$$\widehat{u}_t = y_t - \bar{y} = u_t - \bar{u}.$$

The standard estimator of γ_j is given by

$$\widehat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T} \widehat{u}_t \widehat{u}_{t-j}.$$

Another well known estimator is

$$\hat{\gamma}_j^* = \frac{1}{T-j}\sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j},$$

which is unbiased only if μ is known.

For simplicity and ease of exposition, we focus on the LRV estimator using the Bartlett kernel,

$$\hat{\sigma}^2(S) = \hat{\gamma}_0 + 2\sum_{j=1}^{S-1}(1 - \frac{j}{S})\hat{\gamma}_j,$$

where S is the bandwidth or truncation lag.

2.1. Reduced bias LRV estimator

VY16 defines a linear transformation, labeled as the **A**-matrix approach, based on the mapping between the expectation of the sample autocovariances and the population autocovariances as follows

$$E(\hat{\gamma}) = \mathbf{A}\gamma,$$

where $\hat{\gamma} \doteq [\hat{\gamma}_0\ \hat{\gamma}_1 \cdots \hat{\gamma}_{T-1}]'$, $\mathbf{A} \doteq [a_0\ a_1 \cdots a_{T-1}]'$, $E(\hat{\gamma}_j) = a_j'\gamma$, and $\gamma \doteq [\gamma_0\ \gamma_1 \cdots \gamma_{T-1}]'$. Note that each a_j is a vector of known numbers.

When the population autocovariances of high lags ($\geq M$) are zero (small), exactly (nearly) unbiased estimators of the remaining autocovariances can be obtained using the inverse of upper blocks of the **A** matrix:

$$\widetilde{\gamma}^{(M)} = \left(\mathbf{A}^{(M,M)}\right)^{-1}\widehat{\gamma}^{(M)},$$

where M is the **A**-matrix truncation parameter, $\mathbf{A}^{(M,M)}$ is the upper left $M \times M$ block of **A**, and $\widehat{\gamma}^{(M)}$ is an $M \times 1$ column vector with elements $\hat{\gamma}_0, \hat{\gamma}_1, \ldots, \hat{\gamma}_{M-1}$. $\widetilde{\gamma}^{(M)} \doteq [\widetilde{\gamma}_0^{(M)}\ \widetilde{\gamma}_1^{(M)} \cdots \widetilde{\gamma}_{M-1}^{(M)}]'$ and $\widetilde{\gamma}^{(M)}$ is the **A**-matrix unbiased estimator of $\widehat{\gamma}^{(M)}$. The detailed presentation and calculation of $\mathbf{A}^{(M,M)}$ can be found in VY16.

We can use the **A**-matrix autocovariance estimators $\widetilde{\gamma}^{(M)}$ to construct a bias reduced kernel LRV estimator as

$$\widetilde{\sigma}_M^2(S) = \widetilde{\gamma}_0^{(M)} + 2\sum_{j=1}^{S-1}(1 - \frac{j}{S})\widetilde{\gamma}_j^{(M)},$$

as long as $S \leq M$. Because the **A**-matrix truncation parameter, M, treats higher order autocovariances as approximately zero, it is natural to equate, S, the bandwidth/truncation parameter of $\widetilde{\sigma}_M^2(S)$ with M leading to the Bartlett LRV estimator

$$\widetilde{\sigma}_S^2(S) = \widetilde{\gamma}_0^{(S)} + 2\sum_{j=1}^{S-1}(1 - \frac{j}{S})\widetilde{\gamma}_j^{(S)}.$$

$\widetilde{\gamma}^{(bT)}$ leads to an approximately unbiased LRV estimator for a wide range of b values. In unreported simulations we found this to be true for other kernels including the rectangular kernel. The unbiasedness of $\widetilde{\gamma}_j^{(bT)}$ at almost every lag length j helps reduce the bias in the LRV estimator generated by downweighting from the kernel.

2.2. The LRV estimator of Okui (2010)

Okui (2010) proposed an approximately unbiased autocovariance estimator and used that estimator to propose a kernel LRV estimator. Applying an iterative method Okui (2010) obtained the estimator

$$\hat{\sigma}^{*2}(S) = \left(1 + \frac{\iota_T' K_T}{T - \iota_T' K_T}\right)\hat{\sigma}^2(S)$$

where ι_T is a $T \times 1$ column vectors of ones and in the case of the Bartlett kernel

$$K_T \doteq \left(1, 2(1 - \frac{1}{T})(1 - \frac{1}{S}), 2(1 - \frac{2}{T})(1 - \frac{2}{S}), \ldots, 2(1 - \frac{S-1}{T})(1 - \frac{S-1}{S}), 0, \ldots, 0\right)'.$$

Obviously $\hat{\sigma}^{*2}(S)$ is a scaled version of $\hat{\sigma}^2(S)$ and we show in the Appendix that the scaling factor is asymptotically equivalent to a multiplicative bias adjustment implied by fixed-b theory. Because the scaling constant does not depend on data, the use of $\hat{\sigma}^2(S)$ and $\hat{\sigma}^{*2}(S)$ to construct test statistics leads to equivalent tests when fixed-b critical values are used. Therefore, we only need to compare tests based on $\hat{\sigma}^2(S)$ with tests based on $\widetilde{\sigma}_S^2(S)$.