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An economic "Kessler Syndrome": A dynamic model of earth orbit debris[☆]

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HIGHLIGHTS

- Orbital debris may render orbits economically unprofitable.
- Economic Kessler Syndrome proceeds physical Kessler Syndrome.
- Satellite launch rates respond non-monotonically to debris levels.
- Quantity of debris may increase even in the absence of new satellite launches.
- The model generalizes to any orbit subject to debris accretions or decrements.

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ABSTRACT

We construct a dynamic model of orbital debris that predicts an "economic Kessler Syndrome", where orbital debris renders orbits economically unprofitable, precedes a "physical Kessler Syndrome". Our model generalizes to any orbit subject to debris accretions or decrements.

1. Introduction

In the movie *Gravity*, a Russian missile strike on a decommissioned satellite sets off a collisional cascade between the resultant satellite debris and functioning low-earth orbit satellites, destroying operational satellites, and rendering the orbital space unusable. The science behind the movie is based on work by Kessler and Cour-Palais (1978) who suggest that as debris from launch vehicles and damaged satellites accumulates in orbit, eventually a tipping point is reached where a collisional cascade becomes inevitable. This outcome is popularly referred to as the "Kessler Syndrome", and the National Academy of Sciences reports:

> …the current orbital debris environment [in low-earth orbit] has already reached a "tipping point". That is, the amount of debris, in terms of the population of large debris objects, as well as overall mass of debris currently in orbit, has reached a threshold where it will continually collide with itself, further increasing the population of orbital debris. This increase will lead to corresponding increases in spacecraft failures, which will only create more feedback into the system, increasing the debris population growth rate.[1]

In what follows, we construct a dynamic economic model that suggests an "economic Kessler Syndrome" may precede a "physical Kessler Syndrome", as firms find it economically unprofitable to

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1 National Research Council (2011).

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launch new satellites even when there are no functioning satellites in orbit. We suggest that as the quantity of orbital debris increases, orbital space may become economically unprofitable before it becomes physically unusable.

It is important to note that while Kessler's path-breaking work focuses on low-earth orbit (LEO), the logic and structure of our model generalizes to any orbit subject to accretions or decrements (including gravity and active debris removal). The debris problem is fundamentally one of net accumulation, and the distinction among various orbits is the time scale covering the growth in debris and related collision probabilities. Our model is easily tuned to accommodate any such differences.

Specifically, our model predicts (1) the quantity of space debris will increase even in the absence of new satellite launches when the rate of orbital decay is relatively low, and, (2) launch rates respond non-monotonically to debris: at low levels of debris the relationship is positive and increasing, but at high levels there is a tipping point beyond which launches contract as debris continues to accumulate.

2. Related literature

The economic literature on orbital debris is sparse, and our work represents the first dynamic economic model to capture economic sustainability when launch rates interact with the quantity and rate of decay of orbital debris. Adilov et al. (2014) is in the spirit of this work, but in a two-period model. In an early work, Sandler and Schulze (1981) examine how geostationary space becomes too crowded without station-keeping devices or the ability to move satellites out of the orbital band. They also explore signal interference. Muller et al. (2017) offer a multi-period analysis of satellite launches in the presence of debris. Unlike our effort, they assume (1) exogenous collision probabilities; (2) fixed revenue per satellite; and (3) no launch debris. Within this framework the authors find that satellite operators launch more satellites as debris increases (roughly the opposite of the Kessler effect). Klima et al. (2016) provide a model of debris removal in which two space agencies are engaged in a game and choose debris removal levels. Using a two-period model (Macauley, 2015) investigates a range of policy solutions to debris generation.

3. Model

Assume there is a competitive market providing satellite services, with the associated linear inverse demand function:

$$P_t = a - bS_t \tag{1}$$

where t is an index of time, P_t is price, and S_t is the number of functioning satellites at period t. The total number of satellite launches in period t is L_t. Assume a satellite launch has a constant marginal cost, $h > 0$, and the level of marginal cost of operating a satellite is given by $c > 0$ for a functioning satellite. Finally, assume that $a - c > 0$.

The discount factor is given by β. We assume satellites depreciate at a constant rate δ, where $0 \leq \delta \leq 1$, and depreciated satellites are retired in Earth's atmosphere upon re-entry (in the case of low-earth orbit) or relocated to a "graveyard orbit" (in the case of geostationary satellites).

Let D_t denote the quantity of orbital debris at time t. We assume orbital debris decays at a rate of ϕ, where $0 \leq \phi \leq 1$. Each period, γL_t units of new debris is generated from expended launch vehicles.

Collisions with satellites generate additional orbital debris. We assume a fraction, wD_t, of functioning satellites are hit by debris each period. We think of wD_t as the probability that a satellite might be hit by debris, and this probability increases proportionally with the quantity of debris. We assume a satellite hit by debris becomes non-functioning, and each collision between a satellite and debris generates n units of additional debris. Given this, we write the law of motion for orbital debris as:

$$D_{t+1} = (1-\phi)D_t + \gamma L_t + wnD_t((1-\delta)S_{t-1} + L_t) \tag{2}$$

while the number of functioning satellites in period t is given by:

$$S_t = (1 - wD_t)((1-\delta)S_{t-1} + L_t). \tag{3}$$

Under the assumptions above, orbital space is *physically unusable* at period t if $D_t \geq \frac{1}{w}$, because all functioning satellites will be hit during the period. Thus, $D_t \geq \frac{1}{w} = D^{Kessler}$ is an expression of the "Kessler Syndrome", in which a collisional cascade renders an earth orbit unusable.

Next, we solve the model for the equilibrium number of launches and describe the rate of debris generation. We assume a firm launches a satellite as long as its expected marginal revenue from the launch is not smaller than its expected marginal cost. Expected marginal revenue equals the sum of discounted expected revenue streams from current and future periods and expected marginal cost equals the sum of the firm's discounted expected marginal costs from current and future periods.

To keep the model analytically tractable, firms have adaptive expectations regarding future price levels and probabilities of being hit by orbital debris.[2] In this case, expected marginal revenue from a launch in period t is equal to $(1-wD_t)P_t(1+\beta(1-wD_t)(1-\delta)+\cdots) = \frac{(1-wD_t)P_t}{1-\beta(1-wD_t)(1-\delta)}$, while expected marginal cost equals to $h+(1-wD_t)c(1+\beta(1-wD_t)(1-\delta)+\cdots) = h+\frac{(1-wD_t)c}{1-\beta(1-wD_t)(1-\delta)}$. In equilibrium, expected marginal revenue equals expected marginal cost, i.e., $\frac{(1-wD_t)P_t}{1-\beta(1-wD_t)(1-\delta)} = h + \frac{(1-wD_t)c}{1-\beta(1-wD_t)(1-\delta)}$. Substituting P_t into this equality yields the equilibrium number of satellites and launches in period t:

$$S_t = \frac{(a-c)}{b} - \frac{(1-\beta(1-wD_t)(1-\delta))h}{b(1-wD_t)} \tag{4}$$

$$L_t = \frac{S_t}{1-wD_t} - (1-\delta)S_{t-1}. \tag{5}$$

We note that Eq. (4) assumes that $\frac{(a-c)}{b} - \frac{(1-\beta(1-wD_t)(1-\delta))h}{b(1-wD_t)} \geq (1-\delta)S_{t-1}(1-wD_t)$, i.e., the stock of satellites in period $t-1$ is not too large.

Proposition 3.1. *$L_t = 0$ if $D_t \geq \frac{1}{w}(1 - \frac{h}{a-c+\beta(1-\delta)h})$.*

The proofs of all propositions are given in the Appendix. Proposition 3.1 implies that firms will find it *economically unprofitable* to launch satellites into orbit if the quantity of space debris is larger than a cutoff value (D^{Econ}) even if there are no functioning satellites in orbit (i.e., even when $S_{t-1} = 0$). Comparing this cutoff value to the cutoff value when space becomes physically unusable (the Kessler Syndrome), we see that space becomes economically unprofitable for lower levels of space debris ($D^{Kessler} = \frac{1}{w} > \frac{1}{w}(1 - \frac{h}{a-c+\beta(1-\delta)h}) = D^{Econ}$). In other words, as the quantity of space debris increases, space becomes economically unprofitable before it becomes physically unusable.

Next, we explore changes in space debris over time.

Proposition 3.2. *In equilibrium with $S_t > 0$ and $L_t > 0$, $D_{t+1} - D_t > 0$ if $\phi < \phi^*$ or if $D_t < D^*$ for some $\phi^* > 0$ and $D^* > 0$.*

2 In a follow-on piece, we relax this assumption, and use simulations to solve the model.