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A simple Abel inversion method of interferometric data for temperature measurement in axisymmetric medium

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ABSTRACT

In this work, we describe a simple method of Abel inversion for temperature measurement in a natural convection axisymmetric flow. The essence of the method is that the measured lateral fringe shift profile is fitted with a polynomial with only even powers and then Abel inverse integral is evaluated analytically. This technique is compared with recent existing methods to test the accuracy and error propagation using a simulated interferogram of natural convection flow below a downward-facing heated horizontal disk in air. For this comparison, lateral fringe profiles are simulated using temperature fields computed by solving Navier Stokes and energy equations. Through random-number generation, noise profile is artificially added to the simulated noise-free lateral fringe shift profile. The results showed that the proposed technique for Abel inversion leads to accurate temperature profiles when the lateral fringe shift profile is fitted with even-power polynomials having degrees ranging from 20 to 30.

1. Introduction

In the study of heat transfer between a solid wall and a fluid, temperature measurement in the flow could be an experimental approach to determine the heat transfer coefficient. Currently, it is well accepted that optical interferometry techniques are best suited to measure temperature in natural convection flows, which are known for their high sensitivity to external disturbances. Holographic interferometry [1,2], Mach–Zehnder interferometry [3] and shearing interferometry [4,5] are the most widely used techniques in fluid heat transfer visualization. The principle behind these techniques is rather simple. Temperature and therefore refractive index variations in the medium are converted into fringes patterns that contain both qualitative and quantitative information on heat transfer in the medium. In general, a simple observation of the interferogram provides information on the shape and thickness of the thermal boundary layer [6]. However, when focusing on the quantitative investigations, interferometric diagnostics frequently become difficult. The ease and accuracy of the tomographic reconstruction of the temperature field depend on the symmetry of the studied media. The reconstruction of the cross section of three-dimensional, axially symmetric field is an important problem. In this case the fringes are not isotherms but are contours of the fringe shift defined as the optical path length difference in terms of the used laser wave length. The measured fringe shift from the recorded image of the interferogram is connected to the unknown refractive index along the beam direction by Abel transform. The accuracy of this optical metrology depends strongly on the numerical treatment employed for the Abel inversion, which is highly sensitive to noise close to the centerline. This characteristic leads to great difficulties in performing reliable inversions of experimental data.

Till now, a variety of numerical inversion methods have been published to overcome this problem. Solutions are proposed and tested mainly on measurements in applications such as astronomy, plasma diagnostics and flames studies. Techniques using Fourier–Hankel (FH) and later modified Fourier–Hankel (MFH) or Legendre wavelets (LW) are relatively recent and have been successfully tested in spectroscopy [7,8]. Álvarez et al. [9] compared three Abel-inversion algorithms (discretization method, FH method and Nestor–Olsen method). They found the FH method to be the least sensitive to the number of sample points used and also yielded smaller errors than the other methods. Chan and Hieftje [10] have implemented two methods for computing the Abel inversion calculation, the Nestor–Olsen and the FH algorithms into a LabVIEW program. They reported that the FH method is better (smaller relative residue) than the Nestor–Olsen method only for small numbers of data points. Ma et al. [7] proposed the MFH method by introducing a factor in the numerical expression of the Abel inversion. Adjusting this parameter can significantly increase the precision of this technique. Furthermore, the MFH method has the ability to filter noise

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in the inversion process. In other work, Ma et al. [8,11] have approximated the projected intensity profile by LW. The coefficients of the approximation are computed using the inner product of Legendre wavelets and the intensity profile. The emissivity profile is then obtained by the combination of the functions related to Legendre wavelets. The method was considered more accurate and noise resistant than other methods when applied to experimental data and there is no need for a complete noise filtering of the intensity data before applying Abel inversion.

In this work, we present a simple Abel inversion algorithm for temperature measurement around axisymmetric heated objects in air using interferometric data. We have chosen the case of buoyant convection below a heated horizontal disk facing downward, which was rarely considered [12–14] and deserves more attention for its practical and scientific interest. This problem occurs, for example in studies of chemical vapor deposition CVD process, electronic cooling of compounds and heat transfer from a cylindrical tank.

This paper is organized as follows:

(1) Test profiles of the fringe shift are generated numerically by performing a simulation of temperature fields around a horizontal heated disk in air by solving the coupled Navier–Stokes and energy equations. This approach ensures that test profiles are similar to those obtained experimentally by Chehouani et al. [1].
(2) The proposed method is described together with the MFH and LW methods. These last two methods have never been tested to perform Abel inversion of interferometric data in optical tomography. The three methods will be applied on the generated test profiles of fringe shift to evaluate their accuracy to recover the initial temperature profiles.
(3) In order to determine the sensitivity of these methods to noise, we present the results of their application on the test profiles with artificially additive random noise.

2. Generation of test profiles

Consider a vertical cylinder of 30 mm in diameter and 30 mm in height placed in air as shown in Fig. 1. The term disk refers to the surface facing downward, which is heated to a temperature of 350 °C and the room temperature is $T_0=20$ °C. Buoyancy-driven convection currents induce the establishment of a layer of hot air located in the vicinity of the cylinder. The variation in temperature in this layer leads to a variation of the density and consequently of the index of refraction. The relationship between the refractive index n of air and temperature T can be determined using the ideal gas law and the Gladstone–Dale formula for air [15]. Hence,

$$n = 1 + \frac{KPM}{RT} \tag{1}$$

where $K=2.256\times10^{-4}\,\mathrm{m^3/kg}$, $P=10^5$ Pa, $R=8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ and $M=2.88\times10^{-4}\,\mathrm{kg\,mol^{-1}}$.

The direction of the laser beam, which passes through the thermal boundary layer under the disk, is parallel to the x-axis and perpendicular to the cylinder z-axis. The index of refraction at each cross section only varies in the radial direction. With this assumption, the refractive index function is given by the following expression:

$$f(r) = n(r) - n_0 = \frac{KPM}{R}\left(\frac{1}{T(r)} - \frac{1}{T_0}\right) \tag{2}$$

where n_0 is the refractive index at T_0.

The temperature field neighboring the cylinder is calculated numerically using a mathematical model based on the coupled flow and heat transfer equations [11]. For clarity of presentation, we provide only the result of this simulation. Fig. 2 presents the plots of isotherms from 50 °C to 350 °C with a step of 50 °C. These contours are located very closely to the disk illustrating the

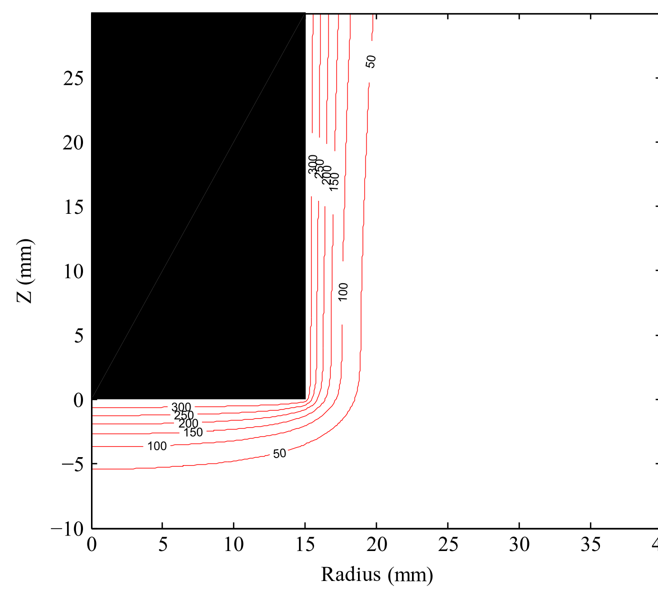

Fig. 2. Isotherms from 50 °C to 350 °C with a step of 50 °C.

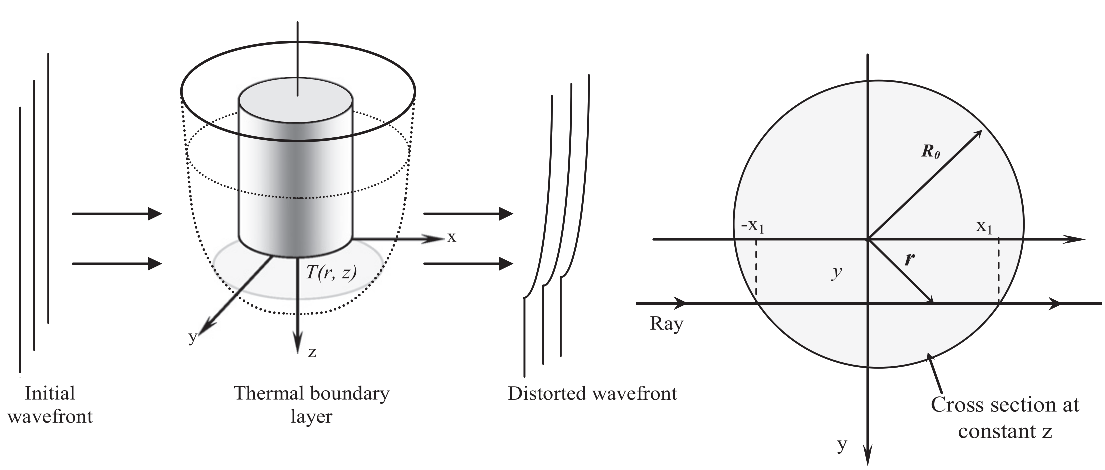

Fig. 1. Schematic of the thermal boundary layer around the heated cylinder and optical path of the test beam.