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On the signal realization set in contracting with information disclosure

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HIGHLIGHTS

- Consider contracting with information disclosure in sense of Bayesian persuasion.
- Question: when can signal realization set be restricted to type set?
- A known result implies conditions under which restriction without loss of generality.
- By means of two examples, show when restriction does entail loss of generality.

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ABSTRACT

I consider contracting with information disclosure in the sense of Bayesian persuasion. A result by Kamenica and Gentzkow (2011) implies that if the principal contracts with a single, uninformed agent, the signal realization set can be restricted to the type set. I show that, otherwise, having additional signal realizations can be advantageous.

1. Introduction

In many monopolistic screening problems, the agents' endowment with private information is not exogenous, but influenced by the principal. Consider for example pricing of new products. Typically, the producer can disclose information that is relevant to the consumers' valuations. Because of idiosyncratic tastes, the consumers may thereby in effect be endowed with *private* information.

Choice over information structures can be modeled as follows. Each agent starts with a noisy prior of his type. The principal also does not know the type, but can design a signal for the agent. Formally, a signal is a random variable that is correlated with the agent's type. The signal realization is only observed by the agent. Afterwards, the agent updates his prior, and is thus endowed with private information. This approach is used in several papers (e.g., Bergemann and Pesendorfer, 2007; Lewis and Sappington, 1994; Li and Shi, 2017).

Choice over information structures is difficult to analyze, because the choice set is very large and complex: every distribution of posteriors can be induced by some signal, with the only restriction that the expected posterior equals the prior (see Kamenica and Gentzkow, 2011). In principle, one must therefore optimize over signals whose realization set has the same cardinality as the set of posteriors. Even if the type set itself is finite, the set of posteriors is uncountable.

In a seminal paper, (Kamenica and Gentzkow, 2011) ("KG") consider the "Bayesian persuasion" problem, in which a sender designs a signal for a single, uninformed receiver who must choose an action. They show that if the type set is finite, signals with more realizations than types cannot be strictly optimal.[1] For a given contract, the problem of designing an optimal signal is an instance

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[1] Their proof is based on a refinement of Caratheodory's Theorem which implies that any feasible expected payoff to the sender (being a convex combination of

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of the Bayesian persuasion problem—provided that the principal contracts with a single, uninformed agent. Hence, KG's result can be applied to contracting with information disclosure.[2]

I show by means of two examples that if the principal contracts with multiple agents or if the agent has private information ex ante, having more signal realizations than types can be strictly optimal.[3] In a nutshell, additional signal realizations for an agent give more flexibility with respect to ex-ante truthtelling constraints of that agent, as well as to truthtelling and participation constraints of other agents.[4]

2. Application of KG's result

Consider a principal and an agent who contract over an allocation $x \in X$. The agent has a type $\theta \in \Theta$, where $|\Theta| < \infty$. Given allocation x and type θ, the agent's payoff is $U(x, \theta)$ and the principal's payoff $V(x, \theta)$.

Neither the principal nor the agent knows the agent's type. They have a common prior $\bar{\mu} \in \Delta(\Theta)$, according to which type θ obtains with probability $\bar{\mu}(\theta) > 0$. Before offering a contract, the principal chooses a signal. A signal, denoted by $\Sigma = (S, (\pi_\theta))$, consists of a finite set S of realizations and probability distributions $\pi_\theta \in \Delta(S)$ such that if type θ obtains, then realization $s \in S$ is drawn with probability $\pi_\theta(s)$. The agent observes the chosen signal and, privately, its realization, and then he updates to a posterior $\mu_s \in \Delta(\Theta)$ according to which type θ obtains with probability

$$\mu_s(\theta) = \frac{\bar{\mu}(\theta)\pi_\theta(s)}{\sum_\theta \bar{\mu}(\theta)\pi_\theta(s)}.$$ [5]

A contract, denoted by $\Gamma = (M, \chi)$, consists of a finite message set M and a function $\chi : M \to \Delta(X)$. After observing the signal realization, the agent accepts or rejects the contract. If he accepts, he must submit a report $m \in M$, upon which the contract implements the distribution of allocations $\chi(m)$. If he rejects, each party takes its outside option, which yields payoff $v(\theta)$ to the principal and payoff $u(\theta)$ to the agent given type θ.

The principal's aim is to design a combination of signal and contract (Σ, Γ) that maximizes her (ex-ante) expected payoff. The problem of designing an optimal signal for a *given* contract is an instance of the Bayesian persuasion problem described by KG. By Proposition 4 of their paper's web appendix, the following holds.

Proposition 1 (*Kamenica and Gentzkow, 2011*). *For every given contract, there exists an optimal signal with $|S| \leq |\Theta|$.*[6]

Return now to the principal's actual problem of designing an optimal signal-contract *combination*. Together with the revelation principle, Proposition 1 allows to restrict attention to signals with $|S| = |\Theta|$ and corresponding direct, incentive-compatible contracts.[7]

payoffs for some collection of posteriors) can be achieved with no more signal realizations than types.

2 KG show that the signal realization set can also be restricted to the receiver's action set. This result extends to multiple receivers and ex-ante private information (see Bergemann and Morris, 2017). In contracting, however, it is not directly applicable, as an agent's action set is endogenous. Indeed, under a direct contract, the action set *is* the signal realization set (ignoring the participation decision).

3 The multi-agent example also delineates KG's result, for which the contract is taken as given. Under ex-ante private information, a fundamental difference to Bayesian persuasion is that the contract can condition on an ex-ante report.

4 That constraints of other agents can be affected is reminiscent of Bester and Strausz (2000) counterexample for their revelation principle with imperfect commitment.

5 Without loss of generality, $\sum_\theta \bar{\mu}(\theta)\pi_\theta(s) > 0$ for all s.

6 The proposition extends to infinite S. Specifically, KG's proof of their Proposition 1 shows that finite signals are sufficient.

7 For a given contract, a contract is direct if $M = S$, and it is incentive compatible if reporting the true signal realization is optimal for the agent.

Table 1
Example 1.

(a) Agent 1's payoffs

	x_A	x_B	x_C	x_D
θ_1	0	−1	−1	0
θ_2	0	1	1	0
θ_3	0	0	0	0

(b) Agent 2's payoffs

x_A	x_B	x_C	x_D
0	1	0	0

(c) Principal's payoffs

	x_A	x_B	x_C	x_D
θ_1	1	1	0	0
θ_2	0	0	1	0
θ_3	0	−1	0	1

(d) Signal Σ^*

	s_A^*	s_B^*	s_C^*	s_D^*
θ_1	1/2	1/2	0	0
θ_2	0	1/2	1/2	0
θ_3	0	0	0	1

3. Use of additional signal realizations

3.1. Multiple agents

Consider the following setting with two agents. Let $X = \{x_A, x_B, x_C, x_D\}$. Agent 1's type lies in $\Theta = \{\theta_1, \theta_2, \theta_3\}$, the prior being $\bar{\mu}(\theta) = 1/3$ for all θ. Agent 2 has a deterministic type. Table 1 gives the agents' and the principal's payoffs.

The principal designs a single direct, incentive-compatible contract to govern the interaction with the agents. Furthermore, she designs a signal for agent 1, whose realization only agent 1 observes. If an agent rejects the contract, each party obtains its outside option. Agent 1's outside option is sufficiently bad such that he will always participate. Agent 2's outside option yields payoff 1/3, and the principal's outside option payoff zero. In the following, I use the notation $\chi(\cdot|s)$ for the distribution over x_A, x_B, x_C, x_D that the contract implements if agent 1 submits report s. Note that incentive compatibility for agent 1 upon any signal realization $s \in S$ requires

$$[\pi_{\theta_2}(s) - \pi_{\theta_1}(s)]\,[\chi(x_B|s) + \chi(x_C|s)] \geq [\pi_{\theta_2}(s) - \pi_{\theta_1}(s)]\left[\chi(x_B|s') + \chi(x_C|s')\right] \quad \forall s' \in S. \tag{1}$$

Now, if the principal knew agent 1's type then she would implement x_B given θ_1 to ensure agent 2's participation, x_C given θ_2, and x_D given θ_3. But if she tries to implement this outcome, agent 1 will deviate and induce x_D if he has type θ_1. Consider the signal-contract combination (Σ^*, Γ^*), given as follows. The signal realization set is $S^* = \{s_A^*, s_B^*, s_C^*, s_D^*\}$. Note that $|S^*| = 4 > |\Theta|$. The distributions π_θ^* over signal realizations are given in Table 1(d). If agent 1 reports s_k^*, $k = A, B, C, D$, the contract Γ^* implements x_k with probability one. It is routine to verify that this contract is incentive compatible for agent 1 and individually rational for agent 2, and that the principal's expected payoff is 5/6.

Take any combination (Σ, Γ) with $|S| = |\Theta| = 3$. By contradiction, suppose the principal's expected payoff, denoted by $\bar{V}$, is at least 5/6. Let $q_\theta = \sum_{s \in S} \pi_\theta(s)\chi(x_B|s)$ be the probability that x_B is implemented given type θ. Individual rationality for agent 2 requires

$$q_{\theta_1} + q_{\theta_2} + q_{\theta_3} \geq 1. \tag{2}$$

Since $\bar{V} \geq 5/6$, it must hold that

$$q_{\theta_2} \leq 1/2 - 2q_{\theta_3} \quad \text{and} \quad q_{\theta_3} \leq 1/4. \tag{3}$$