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A simple test on structural change in long-memory time series

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HIGHLIGHTS

- Simple practitioner's approach to test for change-in-mean under long memory.
- CUSUM test based on the fractionally differenced series.
- Limit distribution is that of the supremum of a standard Brownian bridge.

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ABSTRACT

We propose a simple test on structural change in long-range dependent time series. It is based on the idea that the test statistic of the standard CUSUM test retains its asymptotic distribution if it is applied to fractionally differenced data. We prove that our approach is asymptotically valid, if the memory is estimated consistently under the null hypothesis. Therefore, the well-known CUSUM test can be used on the differenced data without any further modification. In a simulation study, we compare our test with a CUSUM test on structural change that is specifically constructed for long-memory time series and show that our approach performs reasonably well.

1. Introduction

Both long memory and structural change are typical features of macroeconomic and financial time series. Common examples that are considered to possess at least one of these features are inflation rates (cf. Hsu (2005), Bos et al. (2014)), volatilities (cf. Lu and Perron (2010), Xu and Perron (2014), Chiriac and Voev (2011)), unemployment rates (cf. Van Dijk et al. (2002)), and trading volumes (cf. Fleming and Kirby (2011)). A recent review can be found in Gil-Alana and Hualde (2009).

Testing for structural change is important to avoid model misspecification. Unfortunately, this is complicated by the presence of long memory, since long memory and structural change are easily confused. This phenomenon, called spurious long memory, is due to the fact that they can show similar characteristics, such as slowly decaying autocorrelations, or a pole in the periodogram at Fourier frequencies close to zero (see for example Diebold and Inoue (2001) or Granger and Hyung (2004)). To avoid mistaking long memory for structural change, it is therefore necessary to use robust change-in-mean tests that allow for long-range dependence.

One of the most prominent change-in-mean tests is the CUSUM test that was originally proposed by Brown et al. (1975). Here, we focus on the version of Ploberger and Krämer (1992) that is based on OLS residuals, but our results also apply to the recursive version of the test. Wright (1998) and Krämer and Sibbertsen (2002) show that the limiting distribution of the CUSUM test under long memory is different to the short-memory case. This results in the fact that the standard test always rejects asymptotically when the long-memory parameter d is larger than zero. The normalization factor of the CUSUM statistic needs to be larger to account for the slow convergence of the highly persistent long-memory series. Furthermore, the limiting distribution in the short-memory case is based on independent increments, whereas the increments in the long-memory limit distribution are dependent.

A solution is to use another normalization factor and critical values from a fractional Brownian bridge such that the statistic converges to a well defined process and has correct critical values. For Gaussian processes this is done by Horváth and Kokoszka (1997) and extended by Wang (2008) to a wider range of linear

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processes. However, the modified test converges to a non-standard limiting distribution that depends on d.

Recently, Iacone et al. (2013) and Chang and Perron (2016), among others, consider inference about breaks in trends under long memory. Dehling et al. (2013) modify a Wilcoxon-type test for a change-in-mean under long memory by using a consistent estimator of the long run variance. Another group of change-in-mean tests under long memory includes Shao (2011), Iacone et al. (2014), and Betken (2016), who apply a self normalization approach to robustify existing tests.

This paper proposes a different approach as an intuitive alternative—particularly for practitioners. In a two-step procedure we first estimate the long-memory parameter and then perform the standard CUSUM test for structural change on the $\widehat{d}$ times differenced series. This has the advantage that the standard CUSUM test is implemented in most common software packages for statistical analyses. We show that this approach is asymptotically valid. Hence, at least asymptotically, there is no need to use the modified versions of the standard tests.

The rest of the paper is organized as follows. Section 2 describes the model along with the memory robust CUSUM test of Horváth and Kokoszka (1997) and Wang (2008). Section 3 provides and discusses our new test based on differenced data. In Section 4 a Monte Carlo simulation is conducted and Section 5 concludes.

2. CUSUM type tests

We consider a signal-plus-noise model[1] where the observations $(y_t)_{t\geq 1}$ are generated by the stochastic process

$$y_t = \mu_t + \epsilon_t. \tag{1}$$

Here, the regression means $(\mu_t)_{t\geq 1}$ are assumed to be deterministic and fulfill $|\mu_t| < \infty$, for all $t = 1, \ldots, T$. For the error term we assume that $\epsilon_t = \Delta^{-d} v_t$, where v_t is a mean zero martingale difference sequence with finite variance, $|d| < 1/2$, $\Delta^d = (1-L)^d$ is the fractional differencing operator defined as

$$(1-L)^d = \sum_{k=0}^{\infty} \pi_k(d) L^k = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} L^k,$$

and L is the lag operator. That means we consider a process with a possibly time varying mean and stationary fractionally integrated errors ϵ_t. Given a specific time series $y_1, \ldots, y_T$ our interest lies in testing the null hypothesis of a constant unconditional mean

$$H_0: \ \mu_1 = \cdots = \mu_T = \mu$$

against the alternative of a shift in mean

$$H_1: \ \mu_t \neq \mu_s \quad \text{for some} \quad 1 < t \neq s < T.$$

The CUSUM test statistic under long memory (CUSUM-LM) proposed by Horváth and Kokoszka (1997) and Wang (2008) is based on

$$S_T(\tau, \widehat{d}) = \frac{1}{\widehat{\sigma}_c T^{1/2+\widehat{d}}} \left(\sum_{t=1}^{\lfloor \tau T \rfloor} \widehat{\epsilon}_t \right), \quad \tau \in (0, 1), \tag{2}$$

where $\lfloor \cdot \rfloor$ denotes the integer part of its arguments, $\widehat{\sigma}_c$ and $\widehat{d}$ are consistent estimators of the long run variance and the memory parameter and $\widehat{\epsilon}_t$ is the OLS residual from (1). The test statistic is given by

$$Q_T = \sup_{0<\tau<1} |S_T(\tau, \widehat{d})|.$$

For $T \to \infty$ it converges in distribution to the supremum of a fractional Brownian bridge

$$Q_T \Rightarrow \sup_{0<\tau<1} |B_d(\tau) - \tau B_d(1)|,$$

where $B_d(\tau)$ is a fractional Brownian motion and $\Rightarrow$ denotes convergence in distribution.

The statistic also nests the standard CUSUM test for short-memory time series when $d = 0$ and $\widehat{\sigma}_c$ is an autocorrelation consistent (HAC) long run variance estimator (cf. for example Newey and West (1987), or Andrews (1991)). In this case the statistic converges to the supremum of a standard Brownian bridge. Under long memory, σ_c is usually estimated using the MAC estimator of Robinson (2005).

3. CUSUM test after fractional differencing

Instead of using the CUSUM-LM test, we suggest to use the standard CUSUM test on the fractionally differenced series

$$y_t^*(\widehat{d}) = \Delta^{\widehat{d}} y_t = \Delta^{\widehat{d}} \mu_t + \Delta^{\widehat{d}} \epsilon_t = \mu_t^* + v_t^*, \tag{3}$$

where μ_t^* is the fractionally differenced mean, and v_t^* is the fractionally differenced error term, such that $v_t^* = v_t$ in (1), if $\widehat{d} = d$.

Now, denote the partial sum statistic from (2) calculated from the residuals $\widehat{v}_t^*$ and under the assumption that $d = 0$, by $S_T^*(\tau, 0)$. The corresponding CUSUM test statistic is denoted by Q_T^*.

Both approaches, the CUSUM-LM test, as well as ours, require a consistent estimate of d under H_0. Our test requires that for $T \to \infty$ the differenced series $y_t^*(\widehat{d})$ is $I(0)$ and the CUSUM-LM test needs an estimator of d directly in the denominator of the test statistic, and indirectly to obtain the correct critical values. This estimate is usually obtained using the local Whittle estimator of Künsch (1987) and Robinson (1995).

By using fractional differences we implicitly assume that $y_t = \mu_t$ and $\epsilon_t = 0$, for $t \leq 0$, which means that the process was equal to its mean before the beginning of the sample period. This corresponds to a fractionally integrated process of type II. Fractionally integrated processes of type I, on the other hand, assume that ϵ_t has an infinite past. For a detailed discussion of type I and type II fractional Brownian motions see Marinucci and Robinson (1999).

Different from our differencing approach, the CUSUM-LM test usually assumes a process of type I. However, for $0 \leq d < 1/2$, type I and type II processes are asymptotically equivalent.

The following theorem provides the limiting distribution of the test statistic.

Theorem 1. *Suppose that* $\widehat{d} - d_0 = o_P(T^{-\eta})$, *for some arbitrary small* $\eta > 0$ *and* $v_t = 0$ *for all* $t \leq 0$. *Then under* H_0: $Q_T^* \Rightarrow \sup_{0<\tau<1} |B_0(\tau) - \tau B_0(1)|$.

Proof. To prove the theorem, we first show that the partial sum statistic $\tilde{S}_T^*(\tau, 0)$ calculated from the fractionally differenced innovation process v_t^* converges in distribution to $\tilde{S}_T^v(\tau, 0)$ - the statistic based on the true innovation sequence v_t. Denote $\delta = d - \widehat{d}$, then under H_0 with $\epsilon_t = \Delta^{-d} v_t$ and $v_t^* = \Delta^{\widehat{d}} \epsilon_t$ from (1) and (3), we have

$$v_t^* = \Delta^{-\delta} v_t = v_t + \sum_{k=1}^{\infty} \theta_k(\delta) v_{t-k},$$

where $\theta_k(\delta) = \dfrac{\Gamma(k+\delta)}{\Gamma(\delta)\Gamma(k+1)}$.

Therefore, the difference between the fractionally differenced innovation series v_t^* and the original innovation sequence v_t is given by

$$\gamma_t = v_t^* - v_t = \sum_{k=1}^{\infty} \theta_k(\delta) v_{t-k}. \tag{4}$$

[1] One could also assume a standard regression model with $\mu_t = \beta_t' x_t$ and test whether $\beta_t = \beta$.