Economics Letters 163 (2018) 102–105

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Legislative bargaining with a stochastic surplus and costly recognition

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HIGHLIGHTS

- I study legislative bargaining with a stochastic surplus and costly recognition.
- The symmetric stationary payoff is unique under a monotone hazard rate condition.
- For all voting rules, agreement is sooner than that without costly recognition.
- The inefficiency in the timing of agreement increases with the number of agents.

ARTICLE INFO

Article history:
Received 10 September 2017
Received in revised form 26 November 2017
Accepted 8 December 2017
Available online 14 December 2017

JEL classification:
C7
D7

Keywords:
Stochastic bargaining
Costly recognition
Uniqueness
Delays

ABSTRACT

Merging Eraslan and Merlo (2002) and Yildirim (2007), I examine legislative bargaining with a stochastic surplus and costly recognition. I show the uniqueness of the symmetric stationary payoff under monotone hazard rate and an inefficiently early agreement under unanimity.

1. Introduction

Beginning with Baron and Ferejohn (1989), legislative bargaining, where a group of agents divides the surplus according to a majority rule, has received significant attention. Baron and Ferejohn considered an infinitely-repeated sequential bargaining game, in which each period, an agent is recognized randomly to propose a division of the fixed surplus. Yildirim (2007, 2010), and more recently Ali (2015), endogenized the recognition process by introducing a contest to be the proposer. Their extension to competitive recognition builds on a central prediction of the literature: the proposer gets a disproportionate share of the surplus. Merlo and Wilson (1995, 1998) assumed the surplus to be stochastic and analyzed nontrivial agreement delays. In particular, Merlo and Wilson (1998) showed that, under the unanimity rule, equilibrium payoffs are unique, and delays are efficient in that an agreement occurs when the surplus reaches the socially optimal size. Generalizing Merlo and Wilson (1998), Eraslan and Merlo (2002) demonstrated that the uniqueness and efficiency may break down under nonunanimity rules. Specifically, the agreement may be too soon under a nonunanimity rule.

In this paper, I re-examine the uniqueness and efficiency issues by merging the models of Eraslan and Merlo (2002) and Yildirim (2007). Assuming symmetric agents and focusing on the symmetric stationary equilibrium, I show the uniqueness of equilibrium payoff under a monotone hazard rate condition, which is satisfied by many well-known distributions. I also show that costly recognition leads to too early an agreement even under unanimity: to avoid future contests, players are too eager to settle in the present. As expected, the inefficiency in the timing of the agreement grows with the number of agents and the degree of influence on recognition but diminishes with the number of votes required for the agreement.

Next, I briefly present the model and then proceed to its analysis.

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https://doi.org/10.1016/j.econlet.2017.12.013

2. The model

There are n *ex ante* symmetric, risk-neutral agents, who bargain over a stochastic surplus, π, at discrete times $t = 1, 2 \ldots$. I assume that $\pi \in [0, \overline{\pi}]$ is drawn each period independently from a distribution F with a continuous density f. The bargaining protocol is standard: it continues until a proposal receives the consent of at least k agents including the proposer, which I call the *k-majority rule*. I model the recognition process as in Yildirim (2007): if no agreement occurs at $t-1$, time t begins with a Tullock contest in which agents simultaneously exert efforts to be the proposer. Specifically, letting $\mathbf{x}_t = (x_{1t}, \ldots, x_{nt})$ denote the agents' effort profile at t, agent i is recognized according to the following ratio form:

$$p_i(\mathbf{x}_t) = \begin{cases} \dfrac{x_{it}^{\alpha}}{\sum_{j=1}^{n} x_{jt}^{\alpha}} & \text{if} \quad \mathbf{x}_t \neq \mathbf{0} \\ 1/n & \text{if} \quad \mathbf{x}_t = \mathbf{0} \end{cases}, \tag{1}$$

where $\alpha \in (0, 1]$ measures the *degree of influence* on recognition: the higher α, the more efforts influence recognition. For simplicity, I assume a linear effort cost $C(x) = x$.

Following the literature, my bargaining game unfolds as follows. At the beginning of period t, nature draws π. Upon commonly observing π, agents simultaneously choose their efforts, and then the proposer is determined. If an agent is recognized, he either proposes or passes this opportunity. If the proposal is rejected or not put forward, the bargaining moves to the next period, with a new draw of π. Each agent discounts the future by $\delta \in (0, 1)$. As is common in the literature, I focus on the stationary subgame perfect equilibrium in which each agent has a time-independent strategy. As such, I drop the time index below.

3. Analysis and results

In a symmetric equilibrium, let $\widehat{v}_i(\pi)$ be i's expected payoff after observing π but before exerting effort. Then, i's *ex ante* payoff is given by:

$$v_i = E[\widehat{v}_i(\pi)].$$

Note that if recognized, agent i optimally forms a *winning* coalition from only those agents with the lowest continuation values δv_j, i.e., the cheapest votes. Let ψ_{ij} be the probability that agent j is in agent i's winning coalition. Then, agent i's total payment is

$$w_i \equiv \sum_{j \neq i} \psi_{ij} \delta v_j.$$

Upon observing the surplus π, let agent i propose with probability $\sigma_i(\pi)$ and pass this opportunity with probability $1 - \sigma_i(\pi)$, perhaps making an unreasonable offer. Agent i therefore solves the following dynamic program:

$$\begin{aligned} \widehat{v}_i(\pi) = \max_{x_i, \sigma_i(\pi)} \{ & p_i(\mathbf{x})[\sigma_i(\pi)(\pi - w_i) + (1 - \sigma_i(\pi))\delta v_i] \\ & + \sum_{j \neq i} p_j(\mathbf{x})[\sigma_j(\pi)\psi_{ji}\delta v_i + (1 - \sigma_j(\pi))\delta v_i] - x_i \}. \end{aligned} \tag{2}$$

To understand (2), note that with probability $p_i(\mathbf{x})$, agent i is recognized and proposes with probability $\sigma_i(\pi)$. With probability $p_j(\mathbf{x})$, however, agent j is recognized, in which case agent i receives his continuation payoff δv_i if he is in the winning coalition or the proposer causes a delay to next period. (2) implies that, conditional on being the proposer, agent i chooses a simple stopping strategy:[1]

$$\sigma_i(\pi) = \begin{cases} 1, \text{ if } \pi - w_i \geq \delta v_i \\ 0, \text{ if } \pi - w_i < \delta v_i \end{cases}. \tag{3}$$

That is, if recognized, agent i proposes only when he expects to receive a residual surplus greater than his continuation value from passing. Given (1), (2) further implies the following first-order condition for the effort choice:[2]

$$\frac{\partial p_i}{\partial x_i} \times \left\{ [\sigma_i(\pi)(\pi - w_i) + (1 - \sigma_i(\pi))\delta v_i] - \frac{\sum_{j \neq i} p_j[\sigma_j(\pi)\psi_{ji}\delta v_i + (1 - \sigma_j(\pi))\delta v_i]}{1 - p_i} \right\} - 1 = 0, \tag{4}$$

where I drop the argument $\mathbf{x}$ for brevity as I will do in the rest of the paper, and employ the facts that $\frac{\partial p_i}{\partial x_i} = \frac{\alpha}{x_i} p_i (1 - p_i)$ and $\frac{\partial p_j}{\partial x_i} = -\frac{\alpha}{x_i} p_i p_j$ for $j \neq i$.

Using (4), I replace for x_i in (2) to obtain

$$\begin{aligned} \widehat{v}_i(\pi) = \ & \overline{p}_i \times [\sigma_i(\pi)(\pi - w_i - \delta v_i) + \delta v_i] \\ & + (1 + \alpha p_i) \sum_{j \neq i} p_j [\sigma_j(\pi)\psi_{ji}\delta v_i + (1 - \sigma_j(\pi))\delta v_i], \end{aligned} \tag{5}$$

where $\overline{p}_i \equiv (1 - \alpha) p_i + \alpha p_i^2$.

In a symmetric equilibrium, $v_i = v$ and $p_i = \frac{1}{n}$. Hence,

$$w_i = \delta(k-1)v, \quad \overline{p}_i = \frac{1 - \alpha}{n} + \frac{\alpha}{n^2}, \text{ and} \quad \psi_{ji} = \frac{k-1}{n-1}. \tag{6}$$

Substituting (6) into (3) yields

$$\sigma_i(\pi) = \begin{cases} 1, \text{ if } \pi \geq \delta k v \\ 0, \text{ if } \pi < \delta k v \end{cases}. \tag{7}$$

Furthermore, inserting (6) and (7) into (5), and taking expectations of both sides, I obtain the following equation that determines the equilibrium payoff, v.

$$v = \frac{\left[\frac{1-\alpha}{n} + \frac{\alpha}{n^2}\right] \int_{\delta k v}^{\overline{\pi}} [1 - F(\pi)] d\pi}{1 - \delta + \delta \frac{n-k}{n-1} \left[1 - \left(\frac{1-\alpha}{n} + \frac{\alpha}{n^2}\right)\right] [1 - F(\delta k v)]}. \tag{8}$$

Lemma 1 presents the uniqueness result.

Lemma 1. *The symmetric equilibrium payoff, v, is unique under unanimity, $k = n$. Moreover, if $\frac{1 - F(\pi)}{f(\pi)}$ is nonincreasing, it is also unique under any nonunanimity rule, $k < n$.*

Proof. Let $\pi^* = \delta k v$ denote the cut-off surplus under the k-majority rule. Using $\overline{p}_i = \frac{1-\alpha}{n} + \frac{\alpha}{n^2}$, I re-write (8) so that π^* solves:

$$\begin{aligned} \Phi(\pi) \equiv \ & (1 - \delta)\pi - \delta k \overline{p}_i \int_{\pi}^{\overline{\pi}} [1 - F(\widehat{\pi})] d\widehat{\pi} \\ & + \delta \frac{n-k}{n-1} (1 - \overline{p}_i)[1 - F(\pi)]\pi. \end{aligned} \tag{9}$$

The continuity of $\Phi(\pi)$ and the facts that

$$\Phi(0) = -\delta k \overline{p}_i \int_0^{\overline{\pi}} [1 - F(\pi)] d\pi < 0,$$

and

$$\Phi(\overline{\pi}) = (1 - \delta)\overline{\pi} > 0$$

imply the existence of some $\pi^* \in (0, \overline{\pi})$. Moreover, because $\Phi(0) < 0$, a sufficient condition for uniqueness is $\Phi'(\pi^*) > 0$. Differentiating (9) with respect to π, I obtain

$$\begin{aligned} \Phi'(\pi^*) = \ & 1 - \delta + \delta \frac{n - k + (k-1) n \overline{p}_i}{n-1} [1 - F(\pi^*)] \\ & - \delta \frac{n-k}{n-1} (1 - \overline{p}_i) \pi^* f(\pi^*). \end{aligned} \tag{10}$$

[1] If $\pi - w_i = \delta v_i$, agent i is actually indifferent between proposing and passing. However, this is immaterial due to the continuous distribution of π.

[2] The second-order condition is satisfied because, given $\alpha \in (0, 1]$, $\frac{\partial^2 p_i}{\partial x_i^2} \leq 0$.