Economics Letters 162 (2018) 37–40

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

On the equivalence of Bayesian and deterministic dominant strategy implementation☆,☆☆

Srinivas Arigapudi

University of Wisconsin–Madison, Department of Economics, 1180 Observatory Drive Madison, WI 53706, USA

ARTICLE INFO

Article history:
Received 31 July 2017
Received in revised form 18 October 2017
Accepted 27 October 2017
Available online 4 November 2017

JEL classification:
D71

Keywords:
Mechanism design
Bayesian implementation
Dominant strategy implementation
Incentive compatible
Deterministic mechanism

ABSTRACT

In a symmetric single object allocation mechanism with n agents, we identify a necessary and sufficient condition for the existence of an *equivalent* deterministic dominant strategy incentive compatible mechanism for a given Bayesian incentive compatible mechanism.

1. Introduction

In this article we discuss the relationship between Bayesian strategy and dominant strategy implementation of mechanisms in one-dimensional, private independent values setting with linear utilities. We further restrict our attention to a single object allocation mechanism with n agents, in a continuous type space with ex ante symmetric agents and a symmetric allocation rule. In a dominant strategy incentive compatible (DIC) mechanism, each agent's optimal strategy is to report their true type no matter what other players do. When dominant strategies exist, they provide compelling predictions. However, the strong properties required of dominant strategies limits the set of situations where they exist. Negative results on implementation in dominant strategies precipitated a shift to the analysis of Bayesian incentive compatibility. A Bayesian incentive compatible (BIC) mechanism implements the allocation rule in Bayes Nash equilibrium of the game, specifically truth telling always maximizes each agent's interim expected utility. However, in select mechanism design problems there is no loss of generality in imposing the requirement of DIC rather than BIC. For example, in auction environments the auctioneer can create dominant strategy incentives without reducing his expected revenue or altering any of the bidders' interim utilities. When the participants' utilities are unchanged, dominant strategy implementation has certain advantages over Bayesian implementation. For example, in private values environments dominant strategy mechanisms are informationally less demanding than mechanisms that are only BIC. Also dominant strategy equilibria are prior-independent.

Manelli and Vincent (2010) introduced a novel notion of equivalence between any two mechanisms, calling two mechanisms *equivalent* if they deliver the same interim expected utilities to all agents. With independent private values and linear utilities, agents have same interim expected utilities in two mechanisms if and only if every type of every agent has the same expected probability of receiving the object and the same expected transfer under both mechanisms. Most of the earlier literature defines two mechanisms to be *equivalent* if they provide the same ex post allocation. Mookherjee and Reichelstein (1992) showed that the latter form of $BIC - DIC$ equivalence generally fails unless the BIC allocation rule is itself monotonic in each coordinate. In contrast, Manelli and Vincent (2010) construct, for any allocation rule that is Bayesian implementable, another allocation rule that is dominant strategy implementable and that delivers the same interim expected utilities. Gershkov et al. (2013) extend the above result for allocation problems to a wide range of environments which satisfy the assumptions of linear utilities, private and one-dimensional types with independent values.

The mechanism design literature assumes that a mechanism designer can credibly commit to any outcome of a mechanism.

☆ This research is part of my Master's Thesis defended at Indian Statistical Institute, Delhi, India.

☆☆ I am very grateful to Debasis Mishra for his guidance on this project. I wish to thank Sanket Patil and William Sandholm for helpful comments and suggestions.

E-mail address: arigapudi@wisc.edu.

https://doi.org/10.1016/j.econlet.2017.10.026
0165-1765/

This requirement implies that any outcome of the mechanism must be verifiable before it can be employed. Therefore, a stochastic mechanism demands that a randomization device which can be objectively verified be available to the mechanism designer. As noted in Laffont and Martimort (2002), this is not a trivial issue because any deviation away from a given randomization can only be statistically detected once sufficiently many realizations of the contracts have been observed. Such problems do not arise if we have deterministic mechanisms. Many of the well known mechanisms in the literature are *deterministic*.[1] Some common examples include first price auctions, second price auctions and optimal auctions. In this paper, we answer the following question *"Given a symmetric BIC mechanism, under what conditions does there exist an equivalent symmetric deterministic DIC mechanism?"* We show that an equivalent symmetric *DIC* exists if and only if for all types of x, the expected probability of winning the object under the symmetric *BIC* by an agent who reports x is less than or equal to the probability of the remaining agents reporting x.

The organization of the paper is as follows: In Section 2, we introduce the model and characterize *BIC* and *DIC* mechanisms following Myerson (1981). In Section 3, we impose ex ante symmetry of agents and a symmetric allocation rule. In Section 4, we provide necessary and sufficient conditions for the existence of an equivalent symmetric deterministic *DIC* for a given symmetric *BIC* mechanism. In Section 5, we give an example of a symmetric *BIC* mechanism and construct two equivalent symmetric *DIC* mechanisms, one of which is stochastic and the other deterministic.

2. General model

There is a single indivisible object and a finite set $\mathcal{I} = \{1, 2, \ldots, n\}$ of agents. Agent i's type is an element $x_i \in X_i = [0, h_i]$, distributed according to a non-atomic probability distribution F_i with corresponding probability density function f_i. Agents are risk neutral. Preferences are linear in type and money. If t_i is the amount paid by agent i and q_i is the probability that i obtains the object, then i's utility is $x_i q_i - t_i$.

A direct mechanism consists of two functions per agent, $q_i(\mathbf{x})$ and $t_i(\mathbf{x})$, where $q_i(\mathbf{x})$ is the probability that agent i is assigned the object and $t_i(\mathbf{x})$ is the amount i pays when the profile of reports is $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$. Feasibility requires that for all $\mathbf{x}$, we have $\sum_i q_i(\mathbf{x}) \leq 1$.

Fix a mechanism $\{(q_i, t_i)\}_{i\in\mathcal{I}}$. If i reports her type truthfully (and other players report $\mathbf{x}_{-i}$), then i's payoff is $u_i(x_i, \mathbf{x}_{-i}) = q_i(x_i, \mathbf{x}_{-i})x_i - t_i(x_i, \mathbf{x}_{-i})$. Assuming other players also report truthfully, i's expected payoff is $E_{x_{-i}} u_i(x_i) = E_{x_{-i}} q_i(x_i)x_i - E_{x_{-i}} t_i(x_i)$ where

$$E_{x_{-i}} q_i(x_i) = \int_0^{h_1} \ldots \int_0^{h_{i-1}} \int_0^{h_{i+1}} \ldots \int_0^{h_n} q_i(x_i, \mathbf{x}_{-i}) f_1(x_1) \ldots f_{i-1}(x_{i-1}) f_{i+1}(x_{i+1}) \ldots f_n(x_n) d\mathbf{x}_{-i}$$

$$E_{x_{-i}} t_i(x_i) = \int_0^{h_1} \ldots \int_0^{h_{i-1}} \int_0^{h_{i+1}} \ldots \int_0^{h_n} t_i(x_i, \mathbf{x}_{-i}) f_1(x_1) \ldots f_{i-1}(x_{i-1}) f_{i+1}(x_{i+1}) \ldots f_n(x_n) d\mathbf{x}_{-i}.$$

A mechanism is incentive compatible if truthful reporting is an equilibrium. A direct mechanism is *DIC* if reporting truthfully is an equilibrium in weakly-dominant strategies and is *BIC* if reporting truthfully is a Bayesian Nash equilibrium.

Myerson (1981) characterized the sets of *BIC* and *DIC* mechanisms. A mechanism is *DIC* if and only if
(i) for all i and $\mathbf{x}_{-i}$, $q_i(x_i, \mathbf{x}_{-i})$ is non-decreasing in x_i.
(ii) the transfers satisfy

$$t_i(x_i, \mathbf{x_{-i}}) = t_i(0, \mathbf{x_{-i}}) + x_i q_i(x_i, \mathbf{x_{-i}}) - \int_0^{x_i} q_i(y_i, \mathbf{x_{-i}}) dy_i.$$

Similarly, a mechanism is *BIC* if and only if
(i) for all i, $Q_i(x_i) = E_{x_{-i}} q_i(x_i)$ is non-decreasing in x_i.
(ii) the expected transfers satisfy

$$T_i(x_i) = T_i(0) + x_i Q_i(x_i) - \int_0^{x_i} Q_i(y_i) dy_i$$

where $T_i(x_i) = E_{x_{-i}} t_i(x_i)$.

Henceforth we ignore the transfer functions, as they can be recovered up to a constant, from the incentive compatibility conditions.

3. Ex ante identical bidders

We now impose a few restrictions in the general model. We first assume that all agents are ex ante identical: Types are independently and identically distributed according to the non-atomic probability distribution F on $X = [0, h]$. In addition, we require that mechanisms be symmetric, that is, that ex ante identical bidders be treated identically ex ante. In a two bidder case this is equivalent to $q_1(x_1, x_2) = q_2(x_2, x_1)$ for all $x_1, x_2 \in X$. In the most general case, we need some notation to describe symmetry. Let $q : X^n \to [0, 1]$ be such that for every $\mathbf{x} \in X^n$, $\sum_{i=1}^n q(\sigma_i(\mathbf{x})) \leq 1$, where $\sigma_i(x_1, \ldots, x_n) = (x_i, x_2, \ldots, x_{i-1}, x_1, x_{i+1}, \ldots, x_n)$. Each bidder's probability of trade function q_i is derived from the single function q by setting $q_i(\mathbf{x}) = q(\sigma_i(x))$. The symmetric mechanism q can be thus analyzed using the allocation probability function q_i of one of the agents. It is clear that in this case we have $Q_k(.) = Q(.)$ and $F_j(.) = F(.)$ i.e. the probability of trade functions and c.d.f's are the same for all agents. In this setting

(a) q is the allocation function of a symmetric, dominant-strategy incentive compatible mechanism with n bidders if $q(x_1, \mathbf{x_{-1}})$ is nondecreasing in x_1.

(b) q is the allocation function of a symmetric, Bayesian incentive compatible mechanism with n bidders if $E_{\mathbf{x}_{-1}} q(x_1)$ is nondecreasing in x_1.

4. Necessary and sufficient conditions

Our main result shows that given a symmetric *BIC*, there exists an equivalent symmetric deterministic *DIC* if and only if for all types of x, the expected probability of winning the object by an agent who reports x is less than or equal to the probability of the remaining agents reporting less than x.

Theorem 4.1. *If $\tilde{q}$ is a symmetric BIC mechanism with n agents, then there is a symmetric deterministic DIC q that generates the same expected probability of trade i.e. $\widetilde{Q}(x) = Q(x)$ if and only if*

$$\widetilde{Q}(x) \leq F(x)^{n-1} \text{ for all } x \in [0, h]. \quad (1)$$

Proof. First we show sufficiency. Consider the allocation rule given by

$$q_i(x_i, \mathbf{x}_{-i}) = \begin{cases} 1 & x_j < F_j^{-1}\left(\widetilde{Q}(x_i)^{\frac{1}{n-1}}\right) \text{ for all } j \neq i \\ 0 & o.w \end{cases}$$

$\tilde{q}$ is a *BIC* and so has non-decreasing marginals i.e $\widetilde{Q}(x_i)$ is non-decreasing in x_i. Since F_j is a c.d.f, it is non-decreasing and therefore $F_j^{-1}\left(\widetilde{Q}(x_i)^{\frac{1}{n-1}}\right)$ is also non-decreasing. This ensures that $q_i(x_i, \mathbf{x}_{-i})$

[1] A deterministic mechanism is the one in which each $q_i \in \{0, 1\}$, for all agents. Here, q_i is the probability of agent i winning the object.