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# A five-point stencil based algorithm used for phase shifting low-coherence interference microscopy

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ABSTRACT

The phase shifting technique is the most widely used approach for detecting the envelope in low coherence interferometry. However, if the phase shifts calibration contains errors, some parasitic fringe structure will propagate into the calculated envelopes and cause imprecision in the envelope peak detection. To tackle these problems, a five-point stencil algorithm is introduced into the phase shifting interference microscopy. Considering the amount of parasitic fringes, envelope peak detection and computational efficiency, the presented approach leads to satisfactory results in performance. In combination with a simple polynomial curve fitting method the proposed algorithm exhibits good performance on envelope peak detection in surface profiling. Both of the simulated results and the experimental results indicated that the presented approach can be taken as an alternative to the currently existing methods used for phase shifting low-coherence interference microscopy.



## 1. Introduction

Recently, low-coherence interferometry (LCI) has been developed into two new techniques. Coherence probe microscopy (CPM) is used for surface inspection and profiling, particularly in the semiconductor device industry [1]. Optical coherence tomography (OCT) is used for medical diagnostics, particularly in ophthalmology and dermatology [2]. In both these techniques, the broad-band source is often used to locate the envelope peak for CPM and improve the axial resolution for OCT. The short coherence length of the broad-band source is used for optical sectioning properties of these two techniques.

In CPM, interference takes place only from the section of the sample located within the coherence length relative to the reference beam. A series of en-face images is captured by a CCD camera as the sample is scanned axially through focus. Digital filtering techniques [3] have been commonly used to recover the envelope (i.e. fringe modulation) from the sampled data. But this procedure involves two discrete Fourier transforms (forward and inverse) along the $z$ direction for each pixel in the sample and requires the step interval less than a quarter of the shortest wavelength. Consequently this procedure is numerically intensive and time consuming [4]. A more direct approach is based on phase shifting.

The phase shifting technique is originally widely used in phase shifting interferometry (PSI) to extract the phase profile accurately and has been developed to reduce the errors in the phase characterization [5–23], whereas suffers from the $2\pi$ phase ambiguities. However, the phase shifting technique for low-coherence interference microscopy with broad-band source is used to recover the fringe modulation from the intensity information, rather than directly from the phase information as in PSI. Therefore, this technique is implemented via shifting the phase of the reference wave by three or more amounts for each position along the $z$-axis and recording the corresponding values of the intensity. The height of the sample can then be obtained by location of the envelope peak to identify the step nearest to zero optical path difference (OPD) and/or combining this information with the value of the fractional phase [24]. The characterization of the envelopes will be perfect if the phase stepping is perfect. In the event that the phase stepping contains errors, some parasitic fringes will propagate into the calculated envelopes and cause imprecision in the envelope peak detection. Although a large volume of publications involves phase error reduction [5–23], literature on fringe modulation error reduction used for phase shifting low-coherence interference microscopy is hard to come by [25].

This paper presents a less sensitive algorithm used for the phase shifting low-coherence interference microscopy. This approach is derived from the five-point stencil algorithm in numerical analysis. The following paper is arranged as follows. Section 2 introduces the proposed FPS based phase shifting algorithm. Section 3 analyzes and compares the performance of

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the FPS based algorithm with several existing phase shifting algorithms often used for low-coherence interference microscopy. Section 4 confirms the performance of the proposed algorithm experimentally. Section 5 gives some information and discussion about other dominant systematic errors. Section 6 concludes the paper.

## 2. Five-point stencil based algorithm

In numerical analysis, the five-point stencil of a point in the grid is made up of the point itself together with its four "neighbors". It is used to write finite difference approximations to derivatives at grid points. The first, second and third derivatives of a function $f$ at a point $x$ can be approximated using the five-point stencil as [26]

$$f^{(1)}(x)=\frac{f(x-2h)-8f(x-h)+8f(x+h)-f(x+2h)}{12h} \tag{1.1}$$

$$f^{(2)}(x)=\frac{f(x-2h)-16f(x-h)+30f(x)-16f(x+h)+f(x+2h)}{12h^2} \tag{1.2}$$

$$f^{(3)}(x)=\frac{f(x-2h)-2f(x-h)+2f(x+h)-f(x+2h)}{2h^3} \tag{1.3}$$

where $h$ is the spacing between points in the grid. Inspired by the above equations, it is not hard for us to arrive at the idea that $f(x-2h)$, $f(x-h)$, $f(x)$, $f(x+h)$, $f(x+2h)$ might be considered as the five successive frames taken at the phase step value of $h$. Theoretically, the phase step value between frames should be as small as possible to obtain an appropriate approximation to derivative.

In low-coherence interference microscopy, the interference fringe pattern, as shown in Fig. 1, can be considered as a function of $z$ when the sample is scanned in the axial direction:

$$I(x,y,z)=I_0(x,y)+A_i(x,y,z)\times\sin[(2\pi/\lambda_c)\cdot z+\varphi_i(x,y)] \tag{2}$$

where $I_0(x, y)$ is the background intensity variation. $A_i(x, y, z)$, the fringe envelope function, is usually approximated as a Gaussian function for simplification of the calculations [27]. $\sin[(2\pi/\lambda_c)z+\varphi_i(x, y)]$ represents the interference fringes. $\lambda_c$ is the mean wavelength of the source. $\varphi_i$ is the phase step value. $(2\pi/\lambda_c)z$ is equivalent to the unknown (need to know in PSI) phase value, denoted $\phi(x,y)$, at each point in the interferogram. Here coordinates $x$ and $y$ correspond to the transverse object or image coordinates. To determine the fringe envelope function $A_i(x, y, z)$ it is necessary to solve nonlinear Eq. (2) for every point $(x, y)$ of the interference fringe pattern. If the phase step values are properly chosen in advance, there are three unknowns $I_0$, $A_i$, and $\Delta\phi$. Thus, at least three phase shifting frames must be taken to calculate the $A_i(x, y, z)$. However, it may be very difficult to achieve this case in practice. In case phase step values between frames are not determined in advance, at least four fames are necessarily taken to determine the $A_i(x, y, z)$ because of another unknown $\varphi_i$. In the following paragraphs, the coordinates as in Eq. (2) are omitted for simplicity.

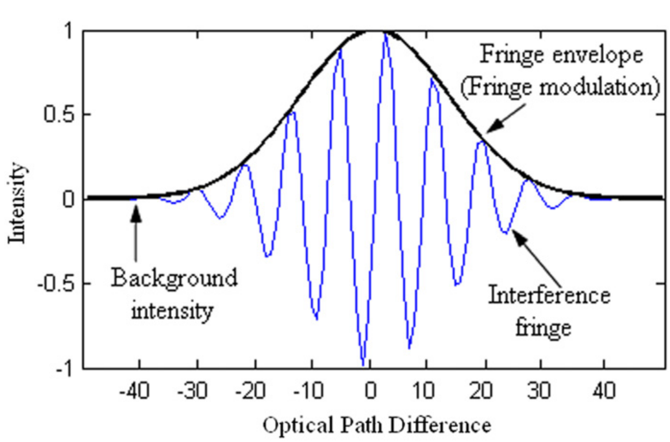


**Fig. 1.** Simulated white light interference fringe and fringe envelope function. The fringe envelope function is a function of the optical path difference between two interference beams, and is usually approximated as a Gaussian function for simplification.

The first, second and third derivatives of $I(x, y, z)$ with respect to shift $z$ can be expressed by

$$I_z^{(1)}=WA_i\cos[Wz+\varphi_i] \tag{3.1}$$

$$I_z^{(2)}=-W^2A_i\sin[Wz+\varphi_i] \tag{3.2}$$

$$I_z^{(3)}=-W^3A_i\cos[Wz+\varphi_i] \tag{3.3}$$

where the constant $2\pi/\lambda_c$ is denoted $W$ for simplification. Combining Eqs. (3.1)–(3.3) and employing the trigonometric formula $\sin^2 a+\cos^2 a=1$, we get

$$A_i^2=(1/W^4)(I_z^{(2)2}-I_z^{(1)2}I_z^{(3)2})$$

$$A_i(x,y)\propto[I_z^{(2)2}-I_z^{(1)2}I_z^{(3)2}]^{1/2} \tag{4}$$

Note $(1/W^4)$ is a constant. According to Eqs. (1.1)–(1.3), the first, second and third derivatives of $I(x,y,z)$ can be expressed by

$$I_z^{(1)}=\frac{I_{-2}-8I_{-1}+8I_1-I_2}{12} \tag{5.1}$$

$$I_z^{(2)}=\frac{I_{-2}-16I_{-1}+30I_0-16I_1+I_2}{12} \tag{5.2}$$

$$I_z^{(3)}=\frac{I_{-2}-2I_{-1}+2I_1-I_2}{2} \tag{5.3}$$

where $I_{-2}$, $I_{-1}$, $I_0$, $I_1$, $I_2$ are five interference fringe images taken at the phase step value $\varphi$. Therefore, Eq. (4) becomes

$$A_i(x,y)\propto\left|\frac{(I_{-2}-16I_{-1}+30I_0-16I_1+I_2)^2}{144}-\frac{(I_{-2}-8I_{-1}+8I_1-I_2)(I_{-2}-2I_{-1}+2I_1-I_2)}{24}\right|^{1/2} \tag{6}$$

The absolute value is to avoid the presence of complex value. Eq. (6) is now referred to as the five-point stencil (FPS) based algorithm with phase step value $\varphi$ between frames.

## 3. Simulations and performance analysis

In order to analyze the performance of the proposed FPS algorithm, several representative algorithms are reviewed before the performance comparisons. The first one is Carré-derived envelope algorithm (denoted A1), which requires four frames with phase step value of an arbitrary constant $2\varphi$. The Carré technique calculates the phase between the steps and uses this information to calculate the modulation [5,16,17,20,23]. The phase step value and the derived fringe modulation or envelope are given by

$$\tan\varphi=\left\{\frac{3[I_2-I_3]-[I_1-I_4]}{I_1-I_4+I_2-I_3}\right\}^{1/2} \tag{7}$$

$$A_i=\left[\left(\frac{I_1-I_4+I_2-I_3}{8\sin\varphi\cos^2\varphi}\right)^2+\left(\frac{I_1+I_4-I_2-I_3}{\cos\varphi\sin^2\varphi}\right)^2\right]^{1/2} \tag{8}$$

The second one (denoted A2) is based on the simplest four-step algorithm utilizing 90° phase steps [20,23,25]. This technique