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Bundling decisions in multi-objects auctions with optimal reserve prices[☆]

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HIGHLIGHTS

- We introduce optimal reserve prices in multi-objects auctions.
- Unbundling can be optimal in two-bidder case with optimal reserve prices.
- Correlation across buyers is crucial in seller's bundling decision.

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ABSTRACT

This paper investigates a seller's bundling decision when he/she sells multiple objects to two potential buyers through auctions in which he/she can optimally set reserve prices. We show how the optimal reserve prices in bundled/separate auctions and the seller's bundling decisions are affected by the correlations of the valuations across buyers and objects. Interestingly, when each object's valuations are sufficiently negatively correlated across buyers, the seller's revenue is higher under unbundling than under bundling. The result is in contrast with that obtained by Palfrey (1983), who showed that, without considering optimal reserve prices, the seller's revenue is always higher under bundling regardless of correlations across either buyers or objects when there are two buyers.

1. Introduction

A seller has to decide whether to bundle multiple objects together when he/she sells these objects to potential buyers through auction. One of the key results in the literature of multi-objects auction is that the seller's bundling decision is strongly influenced by the number of buyers (Palfrey, 1983; Chakraborty, 1999, 2006; Jehiel et al., 2007). The seller is more likely to choose bundling if there are few buyers. In particular, Palfrey (1983) showed that bundling is always optimal for the seller if there are only two buyers, regardless of the dependence of the buyers' valuations across bidders or objects.

These papers, however, fail to take into account that the seller can optimally set reserve prices in auctions. In their set-up, the sellers are not allowed to set positive reserve prices, and trade happens with probability of one even if buyers' bids are extremely low. For the private values and one-object case, Myerson (1981) and Riley and Samuelson (1981) found that zero reserve price is not optimal for a revenue-maximizing seller. In fact, they proved that the optimal auction is a second-price auction with a positive reserve price. It is therefore legitimate to ask how the possibility of allowing the seller to set optimal reserve prices changes the seller's bundling decisions.[1]

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[1] The model in Jehiel et al. (2007) also allows for reserve prices, but does not explicitly study the relation of optimal reserve prices and the bundling decision. In

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This paper studies the seller's bundling decision when he/she sells two objects to two potential buyers through second-price auctions with reserve prices. Buyers have private and additive valuations for the two objects. For simplicity, we assume that buyers' valuations for objects are binary: A buyer's valuation for an object is either high (v_h) or low (v_l). We allow for buyers' valuations between bidders and goods to be correlated.

We first show how the optimal reserve prices are affected by the correlations of buyers' valuations across bidders and goods. Then, we show how the possibility of allowing the seller to optimally choose reserve prices changes the seller's bundling decision. Interestingly, unbundling can be optimal for the seller even if there are only two buyers, which is in contrast to the result of Palfrey (1983). The condition that valuations are sufficiently negatively correlated across buyers is crucial for unbundling to be optimal. The intuition can be easily seen in the extreme case, where valuations are perfectly negatively correlated both across bidders and across objects. In such an extreme case, the seller's revenue in the bundled auction is $v_l + v_h$. His/her revenue is $2v_h$ if he/she optimally set the reserve price in the two separate auctions to be v_h. Our result suggests that the possibility of allowing the seller to optimally choose reserve prices helps unbundling to gain more advantage over bundling from the perspective of the seller.

2. The model

A risk neutral seller has one unit of each of 2 indivisible goods to sell. The seller's cost for each good is zero. There are two risk-neutral buyers. Let x_i^j be buyer i's holding (either 0 or 1) of good j, where $i, j = 1, 2$. Buyer i has a utility function $U^i = M^i + x_i^1 v_i^1 + x_i^2 v_i^2$, where v_i^j is the value of item j to buyer i and M^i is buyer i's wealth.

There are four random variables in our model: $v_1^1, v_1^2, v_2^1, v_2^2$. Assume $v_i^j \in \{v_l, v_h\}$, where $i, j = 1, 2$ and $v_l < v_h$. The joint distribution of the four variables are assumed to be symmetric, which gives that $\Pr\left(v_i^j = v_l\right) = \Pr\left(v_i^j = v_h\right) = \frac{1}{2}, \forall i, j = 1, 2$. In addition, we have the following definitions[2] :

$$\alpha = \Pr\left(v_{i'}^j = v_l | v_i^j = v_l\right) = \Pr\left(v_{i'}^j = v_h | v_i^j = v_h\right),$$

$$\beta = \Pr\left(v_i^{j'} = v_l | v_i^j = v_l\right) = \Pr\left(v_i^{j'} = v_h | v_i^j = v_h\right),$$

$$p = \Pr\left(v_i^j = v_k, v_i^{j'} = v_k, v_{i'}^j = v_k, v_{i'}^{j'} = v_k\right).$$

The parameter α is understood as the correlation across buyers. That is, it characterizes the correlation between different buyers' valuations of the same good. Specifically, $\alpha < \frac{1}{2}$ means that the valuations of each object are negatively correlated across buyers; $\alpha > \frac{1}{2}$ means that the valuations of each object are positively correlated across buyers; $\alpha = \frac{1}{2}$ means that the valuations of each object are independent across buyers. Similarly, parameter β can be understood as the correlation across objects, with $\beta < \frac{1}{2}$ specifying that each buyer's valuations are negatively correlated across objects; $\beta > \frac{1}{2}$ specifying that each buyer's valuations are positively correlated across objects, and $\beta = \frac{1}{2}$ meaning that each buyer's valuations for the two goods are independent. Finally, the parameter p is the probability that valuations are the same both across buyers and across objects.

The auction mechanism considered here is the second-price (Vickrey) auction with reserve price. In the Vickrey auction, bidding their true valuations are bidders' weakly dominating strategies. The seller's objective is to maximize his/her expected profit, by choosing from the following two procurement strategies: 1) Unbundling. The seller sells the two goods in two separate auctions; 2) Bundling. The seller bundles the two goods together and bidders bid for the bundled goods. Unlike Palfrey (1983), we assume that the seller can optimally set a reserve price in an auction, i.e., a bidder can win the object only when his/her bid exceeds the reserve price. If the seller chooses unbundling, he/she sets two reserve prices for each of the two goods. If the seller chooses bundling, he/she sets one reserve price for the bundled product. In the second-price auction with reserve price, there are three possible cases: 1) No bid exceeds the reserve price. In this case, no one wins the object. 2) Both bids exceed the reserve price. In this case, the winner pays the second highest bid. 3) One bid exceeds the reserve price while the other bid does not. In this case, the bidder with the highest bid wins the object and pays the reserve price.

3. Analysis: unbundling vs. bundling

3.1. Unbundling

Suppose the seller chooses unbundling. Because of our symmetric assumption, we only need to analyze the auction for good 1, and the analysis for the auction for good 2 is exactly the same. In the auction for good 1, the optimal reserve price should be either 0[3] or v_h. Suppose the seller sets a reserve price 0. Then, the seller's profit is v_h if both buyers' valuations are v_h, and v_l in other cases. The seller's expected profit from auctioning 1 is hence equal to $\pi_u^1(0) = \frac{1}{2}\alpha v_h + \left(1 - \frac{1}{2}\alpha\right) v_l$, where the subscript u indicates unbundling, and superscript 1 refers to good 1. Suppose the seller sets a reserve price v_h. Then, the seller obtains v_h if at least one of the buyers' valuation is v_h, and 0 if both buyers' valuations are v_l. The seller's expected profit is equal to $\pi_u^1(v_h) = \left(1 - \frac{1}{2}\alpha\right) v_h$. Comparing $\pi_u^1(v_h)$ and $\pi_u^1(0)$, we know that the sell will optimally set the reserve price equal to v_h if $\alpha \le \hat{\alpha} = \frac{2v_h - 2v_l}{2v_h - v_l}$. Notice that $0 < \hat{\alpha} < 1$. The seller's expected profit is given by $\pi_u(v_h) = 2\pi_u^1(v_h)$ and $\pi_u(0) = 2\pi_u^1(0)$. The seller's optimal profit is $\pi_u^* = \max(\pi_u(v_h), \pi_u(0))$. We summarize our results in the following proposition:

Proposition 1. *Suppose the seller chooses unbundling. Then, in both separated auctions, the seller's optimal reserve price is equal to v_h if $\alpha \le \hat{\alpha} = \frac{2v_h - 2v_l}{2v_h - v_l}$, and 0 if $\alpha > \hat{\alpha}$. The seller's total profit is given as follows*

$$\pi_u^*(\alpha) = \begin{cases} (2-\alpha) v_h & \text{if } \alpha \le \hat{\alpha} \\ \alpha v_h + (2-\alpha) v_l & \text{if } \alpha > \hat{\alpha} \end{cases}.$$

The above proposition suggests that the seller is more likely to set a reserve price if the correlations of the valuations across buyers are negative. The intuition is as follows. A higher reserve price means a higher revenue once trade happens, but a smaller probability of trading. The trade will not happen if both buyers' valuations are low. If the valuations of each object are more negatively correlated across buyers, the probability that both buyers have low valuations is smaller. Thus, it is more likely that the seller will choose a higher reserve price.

a context of bundled procurement, Chen and Li (2015) also consider positive reserve prices, but the reserve prices are exogenously given instead of endogenously chosen by the auctioneer.

2 For any symmetric joint distribution, we must have that (i) $2p \le \alpha$; (ii) $2p \le \beta$; and (iii) $\alpha + \beta - 2p \le 1$. Throughout this paper, we only consider parameters α, β, and p such that these conditions are satisfied.

3 Any reserve prices which are smaller or equal to v_l are equivalent.