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Product and process innovation with knowledge accumulation in monopoly: A dynamic analysis

Genyuan Zhong, Weihang Zhang *

Antai College of Economics and Management, Shanghai Jiaotong University, HuaShan Road 1954, Shanghai, 200030, China

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ABSTRACT

In this paper, we investigate monopolist optimal investment levels using a dynamic model under knowledge accumulation. We show that (i) investment decisions in product and process innovation are independent of each other; (ii) under both the monopolist and the social optimum, there exist the saddle stable steady state equilibria; (iii) Optimal investment levels in product and process innovation positive response to learning rate and knowledge accumulation rate, inversely response to knowledge depreciation rate.

1. Introduction

In this paper, we use a dynamic method to address the optimal control problem of product and process innovation with knowledge accumulation. Especially, we introduce the depreciation characteristic of knowledge and identify the direct effects of knowledge accumulation on product quality and production cost. Furthermore, knowledge accumulation under investment is assumed to appreciate at a constant rate and knowledge is continuously depreciating at a constant rate as the old knowledge replaced by the new one. We focus on a monopolistic market where a firm is protected by a patent and prices its products dynamically. Our main aim is to study investment decisions under knowledge accumulation, so here we use a specially fixed demand curve for simplicity and to be intuitionistic. What is more, inelastic demand is common in housing market, gasoline market and luxury goods market. So conclusions drawn in our article are especially meaningful to such industries.

The main contribution of our paper can be concluded as three aspects. First, we extend the model of Lambertini and Orsini (2015) into a more general one with characteristics of knowledge accumulation. In our model, the cost functions of product and process innovation depend on both instantaneous investments and knowledge accumulation. Second, while Li and Ni (2016) developed the optimal control model with learning-by-doing, we investigate both the knowledge accumulation rate under investment and the knowledge depreciation rate against the background of quickly changing technology using a rigid demand curve. In our model, the state functions of knowledge accumulation include elements of knowledge accumulation rate and knowledge depreciation rate. Furthermore, state functions of product quality and process quality include not only the effects of investment, but also the direct effects of knowledge accumulation.

The rest of the paper is organized as follows: the basic model of dynamic optimal control with knowledge accumulation is introduced in Section 2 after the introduction. Then the equilibrium analysis is made in Section 3. In Section 4 we do stability analysis of the Jacobian matrix. Finally, we discuss our conclusions in Section 5.

2. The basic model

In this paper, we consider a single-product monopoly selling a piece of nondurable goods of quality $q(t) > 0$ at price $p(t) > 0$ over continuous time $t \in [0, +\infty)$. Here according to Lambertini and Orsini (2015) we assume the level of marginal willingness to pay for quality θ is uniformly distributed with a density $d(d \geq 1)$ and $\theta \in [\Theta - 1, \Theta]$, where $\Theta > 1$. Under the full market coverage assumption, an individual who buys a single unit of goods has net surplus:

$$U = \theta q(t) - p(t) \geq 0 \tag{2.1}$$

* Correspondence to: Panyu Road 655, Shanghai, 200030, China.
E-mail addresses: genyuanzh@sjtu.edu.cn (G. Zhong), coccinellin@sjtu.edu.cn (W. Zhang).

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As is proposed by the assumption, the poorest consumer will buy the product under profit-maximization price, i.e.,

$$D(t) = x(t) = d \text{ and } p^m(t) = (\theta - 1) q(t) \tag{2.2}$$

At any instant, the monopoly chooses investment levels of product and process innovation with knowledge accumulation. Instantaneous investment in product innovation increases cumulative product quality while instantaneous investment in process innovation decreases the cumulative cost of production (Chenavaz, 2012). Differential equations describing the time evolution of the product quality and production costs are given as

$$\dot{q}(t) = k(t) - \delta q(t) \tag{2.3}$$

$$\dot{c}(t) = -h(t) + \sigma c(t) \tag{2.4}$$

in which $\delta > 0$ is the decay rate of quality while $\eta > 0$ is the obsolescence rate affecting production technology. According to Thompson (2010), knowledge accumulations of product innovation $A_1(t)$ and process innovation $A_2(t)$ during time $t \in [0, +\infty)$ can be defined as:

$$A_1(t) = A_{10} + m_1 \int_0^{+\infty} k(s) ds \tag{2.5}$$

$$A_2(t) = A_{20} + m_2 \int_0^{+\infty} h(s) ds \tag{2.6}$$

with the rapid development of society and technology, the stock of old knowledge decays as new ideas taking the place of old ones. As suggested by Jorgenson (1973) and Griliches (1998), a proportional or geometric depreciation rule seems to be a good choice to represent the depreciation of aggregate stock of knowledge. To simplify, we apply the proportional and linear approach using n_1, n_2 to represent depreciation rate of the aggregate stock of knowledge evolving over time. Taking knowledge depreciation into consideration, Eq. (2.5) and (2.6) in our model are revised as follows:

$$A_1(t) = A_{10} + m_1 \int_0^{+\infty} k(s) ds - n_1 \int_0^{+\infty} A_1(s) ds \tag{2.7}$$

$$A_2(t) = A_{20} + m_2 \int_0^{+\infty} k(s) ds - n_2 \int_0^{+\infty} A_2(s) ds \tag{2.8}$$

Here we interpret parameters $m_1, m_2 > 0$ as rate of knowledge accumulation under investment in product and process innovation, and parameters $n_1, n_2 > 0$ as depreciation rate of knowledge $n_1, n_2 > 0$.

Besides, accumulation of knowledge can not only improve the cumulative product quality but also reduce the cumulative cost of production directly. So in our new-developing model the differential equations are given by the following new forms:

$$\dot{q}(t) = k(t) + \mu_1 A_1(t) - \delta q(t) \tag{2.9}$$

$$\dot{c}(t) = -h(t) - \mu_2 A_2(t) + \sigma c(t) \tag{2.10}$$

$$\dot{A_1}(t) = m_1 k(t) - n_1 A_1(t) \tag{2.11}$$

$$\dot{A_2}(t) = m_2 h(t) - n_2 A_2(t) \tag{2.12}$$

where $\mu_1, \mu_2 > 0$ are constant coefficient.

According to the model of product-process innovation with learning by doing (Li and Ni, 2016), the monopoly's cost functions of product and process innovation are given by the following forms:

$$C(k(t), A_1(t)) = \alpha k^2(t) - b_1(A_1(t) - A_{10}) \tag{2.13}$$

$$C(h(t), A_2(t)) = \beta h^2(t) - b_2(A_2(t) - A_{20}) \tag{2.14}$$

where $C(k(t), A_1(t))$ and $C(h(t), A_2(t))$ are increasing with $k(t)$, $h(t)$, decreasing with $A_1(t)$, $A_2(t)$. We define $b_1, b_2 > 0$ as the learning rates of product innovation and process innovation.

Therefore the monopolist's instantaneous profits are

$$\begin{aligned} \pi(t) = {} & [(\theta - 1) q(t) - c(t)] d - \left[\alpha k^2(t) - b_1(A_1(t) - A_{10})\right] \\ & - \left[\beta h^2(t) - b_2(A_2(t) - A_{20})\right] \end{aligned} \tag{2.15}$$

As the objective of monopolist is to find the optimal levels of investment in product and process innovation to maximize the discounted profit flow. Combining the dynamic Eqs. (2.9)–(2.12) with the profits' expression, we can obtain the model of product and process innovation under knowledge accumulation:

$$\begin{aligned} \Pi = \max_{k,h} \int_0^{+\infty} & e^{-rt} \{[(\theta - 1) q(t) - c(t)] d \\ & - \left[\alpha k^2(t) - b_1(A_1(t) - A_{10})\right] \\ & - \left[\beta h^2(t) - b_2(A_2(t) - A_{20})\right]\} dt \end{aligned}$$

$$\begin{cases} \dot{q}(t) = k(t) + \mu_1 A_1(t) - \delta q(t) \\ \dot{c}(t) = -h(t) - \mu_2 A_2(t) + \sigma c(t) \\ \dot{A_1}(t) = m_1 k(t) - n_1 A_1(t) \\ \dot{A_2}(t) = m_2 h(t) - n_2 A_2(t) \end{cases} \tag{2.16}$$

In this model, the control variables are investment in product innovation $k(t)$ and investment in process innovation $h(t)$; the state variables are product quality $q(t)$, production cost $c(t)$, change rate of knowledge accumulation in product innovation $A_1(t)$ and change rate of knowledge accumulation in process innovation $A_2(t)$. Profits are discounted at a constant rate $r > 0$.

3. Equilibrium analysis

The Hamilton function is represented by H and let $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$ and $\lambda_4(t)$ be the dynamic costate variables associated with their respective states equation $\dot{q}(t)$, $\dot{c}(t)$, $\dot{A_1}(t)$ and $\dot{A_2}(t)$ which are evaluated at time t, respectively. The corresponding current Hamiltonian function of (2.16) is:

$$\begin{aligned} H = {} & [(\theta - 1) q(t) - c(t)] d - \left[\alpha k^2(t) - b_1(A_1(t) - A_{10})\right] \\ & - \left[\beta h^2(t) - b_2(A_2(t) - A_{20})\right] \\ & + \lambda_1(t) [k(t) + \mu_1 A_1(t) - \delta q(t)] \\ & + \lambda_2(t) [-h(t) - \mu_2 A_2(t) + \sigma c(t)] \\ & + \lambda_3(t) [m_1 k(t) - n_1 A_1(t)] \\ & + \lambda_4(t) [m_2 h(t) - n_2 A_2(t)] \end{aligned} \tag{3.1}$$

To maximize Π (2.16), the first-order conditions of current Hamiltonian function are the following:

$$\frac{\partial H}{\partial k(t)} = -2\alpha k(t) + \lambda_1(t) + m_1 \lambda_3(t) = 0 \tag{3.2}$$

$$\frac{\partial H}{\partial h(t)} = -2\beta h(t) - \lambda_2(t) + m_2 \lambda_4(t) = 0 \tag{3.3}$$

The costate equations are shown as follows:

$$\dot{\lambda_1}(t) = r\lambda_1(t) - \frac{\partial H}{\partial q(t)} = (r + \delta) \lambda_1(t) - (\theta - 1) d \tag{3.4}$$

$$\dot{\lambda_2}(t) = r\lambda_2(t) - \frac{\partial H}{\partial c(t)} = (r - \sigma) \lambda_2(t) + d \tag{3.5}$$

$$\dot{\lambda_3}(t) = r\lambda_3(t) - \frac{\partial H}{\partial A_1(t)} = (r + n_1) \lambda_3(t) - \mu_1 \lambda_1(t) - b_1 \tag{3.6}$$

$$\dot{\lambda_4}(t) = r\lambda_4(t) - \frac{\partial H}{\partial A_2(t)} = (r + n_2) \lambda_4(t) + \mu_2 \lambda_2(t) - b_2 \tag{3.7}$$

The transversality conditions of the differential equations (3.4)–(3.7) are $\lim_{t\to\infty} \lambda_1(t) q(t) e^{-rt} = 0$, $\lim_{t\to\infty} \lambda_2(t) c(t) e^{-rt} = 0$, $\lim_{t\to\infty} \lambda_3(t) A_1(t) e^{-rt} = 0$ and $\lim_{t\to\infty} \lambda_4(t) A_2(t) e^{-rt} = 0$.

The concomitant variables $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$ are shadow prices related to $q(t)$, $c(t)$, $A_1(t)$, $A_2(t)$ as was discussed by Heckman (1974), respectively.