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A new tight and general bound on return predictability

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HIGHLIGHTS

- I propose a novel upper bound on the predictability of asset excess-returns.
- The novel bound is tighter than the bound proposed by Ross (2005) and at least as tight, or tighter, than the latter.
- The novel bound also holds in a broader set of circumstances than the bound put forth by Huang and Zhou (2017).
- I demonstrate the use of the bound by testing for the efficiency of the currency market.

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ABSTRACT

We propose a novel upper bound on the predictability of asset returns. This bound is tighter than the bound proposed by Ross (2005) because it takes into account not only the volatility of the pricing kernel but also the correlation between the pricing kernel and trading strategies that exploit predictability. It is also at least as tight as the bound proposed by Huang and Zhou (2017). We apply our bound to study the predictability of returns on currencies of emerging and developed economies from 1994 to 2016. We find evidence of return predictability in excess of the bound, especially for emerging markets currencies. This implies either market inefficiency or, alternatively, that investors either can become very risk-averse or price currencies using a model radically different from the CAPM. In contrast, the evidence of excess-predictability is much weaker under the wider bound proposed by Ross (2005).

1. Introduction

The upper bound on the predictability of asset excess-returns in efficient markets put forth by Ross (2005) is the following:

$$R^2 \leq \phi \equiv (1+R_f)RRA_V{}^2\sigma^2(r_M) \approx RRA_V{}^2\sigma^2(r_M). \tag{1}$$

Here, R^2 is the coefficient of determination of any predictive model of asset excess-returns, R_f is the risk-free rate of return (assumed constant),[1] RRA_V is a relative risk aversion (RRA) upper bound, and $\sigma(r_M)$ is the volatility of the excess-return, r_M, on portfolio M, which is held by investors, to whom Ross (2005) refers as the "sharks", who have the capability to exploit the asset return predictability.[2] This bound captures a key implication of the Efficient Market Hypothesis (EMH) in an elegantly succinct way but, as noted by Huang and Zhou (2017), it is too wide to meaningfully restrict predictability in most applications.

In the next section, we derive a bound that generalizes and tightens the bound proposed by Ross (2005) and we compare it with the bounds proposed by Zhou (2010) and Huang and Zhou (2017). In the subsequent two sections, we present an empirical application of our bound to testing the efficiency of the currency market. In the final section, we offer our conclusions and outline future research avenues.

2. Bounding predictability

Consider a general predictive model that uses information given by the filtration $\{I_t\}_{t\geq 0}$, i.e.

$$r_{t+1} = \bar{\mu} + (\mu(z_t) - \bar{\mu}) + \epsilon_{t+1}. \tag{2}$$

Here, $\bar{\mu} \in \mathbb{R}$ is a constant, $z_t \in I_t$, $\mu(z_t) := E(r_{t+1}|I_t)$ is the forecast that uses the information set I_t and the error, ϵ_{t+1}, is unpredictable with respect to I_t, so that $E(\epsilon_{t+1}|I_t) = 0$.

For a given risky asset and a given predictive model of r_{t+1}, an upper bound on the model R^2 is then given in the following Proposition.

Proposition 1. *Assume that the EMH holds and m_{t+1} is a kernel that prices all asset excess-returns in the economy, including those*

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[1] The second approximate equality holds for realistically small values of R_f.

[2] In empirical applications, Ross (2005) suggests that a good proxy for this portfolio is one resembling the S&P 500, taken to exemplify a well diversified portfolio which is on, or is close to, the mean–variance frontier of risky assets.

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on dynamic strategies. Then, for the excess-return on any given risky asset, the coefficient of determination of a given predictive model like (2) is bounded from above as follows:

$$R^2 \leq \phi \equiv \rho^2(s_{t+1}, m_{t+1})\sigma^2(m_{t+1}). \tag{3}$$

Here, $\rho(s_{t+1}, m_{t+1})$ denotes the unconditional correlation between s_{t+1} and m_{t+1}, s_{t+1} is defined as follows

$$s_{t+1} := r_{t+1}(\mu(z_t) - \bar{\mu}) \tag{4}$$

and $\sigma^2(m_{t+1})$ is the unconditional variance of m_{t+1}.

Proof. We have that $R^2 \equiv \frac{\sigma^2_\mu}{\sigma^2_r}$, with $\sigma^2_\mu := \sigma^2(\mu(z_t))$ and $\sigma^2_r := \sigma^2(r_{t+1})$. The R^2 of the predictive model can, therefore, be decomposed as follows:

$$R^2 \equiv \frac{\sigma^2_\mu}{\sigma^2_r} = \frac{E(\mu^2(z_t)) - (E(\mu(z_t)))^2}{\sigma^2_r}. \tag{5}$$

Since the asset is (by assumption) risky, $\sigma_\epsilon := \sigma^2(\epsilon_{t+1}) = E(\sigma^2(\epsilon_{t+1}(z_t))) > 0$, where $\sigma^2(\epsilon_{t+1}(z_t)) := E(\epsilon^2_{t+1}|z_t)$, and $0 \leq R^2 < 1$. Hence, we can re-write (5) as follows:

$$\begin{aligned} R^2 &= \frac{E\left(\mu^2(z_t)\right)}{\sigma^2_\epsilon/(1-R^2)} - \frac{(E(\mu(z_t)))^2}{\sigma^2_r} = \frac{E\left(\mu^2(z_t)\right)}{\sigma^2_\epsilon}(1-R^2) - \frac{(E(r_{t+1}))^2}{\sigma^2_r} \\ &= \frac{E\left(\mu^2(z_t)\right)}{E(\sigma^2_\epsilon(z_t))}(1-R^2) - \frac{(E(r_{t+1}))^2}{\sigma^2_r} \\ &\leq E\left(\frac{\mu^2(z_t)}{\sigma^2_\epsilon(z_t)}\right)(1-R^2) - \left(\frac{E\,(r_{t+1})}{\sigma_r}\right)^2. \end{aligned}$$

Here, the above inequality arises because of Jensen's inequality, since any ratio of positive quantities is a convex function of the denominator. Hence, we have

$$R^2 \leq E\left(\left(\frac{\mu(z_t)}{\sigma_\epsilon(z_t)}\right)^2\right)(1-R^2) - \left(\frac{E(r_{t+1})}{\sigma_r}\right)^2.$$

The expression inside the expectation in the first term on the right-hand side of this inequality is the square of the conditional SR, $SR_t(r_{t+1}) := \frac{\mu(z_t)}{\sigma_\epsilon(z_t)}$, whereas the second term is simply the square of the unconditional SR, $SR(r_{t+1}) := \frac{E(r_{t+1})}{\sigma_r}$, attainable by holding the asset. We can thus write:

$$R^2 \leq E\left(SR^2{}_t\,(r_{t+1})\right)\left(1-R^2\right) - SR^2\,(r_{t+1}). \tag{6}$$

Since the EMH holds, m_{t+1} is a kernel that prices all asset excess-returns in the economy, including those on dynamic strategies, and I_t is the information available to market participants at time t, it must be that[3]

$$E(m_{t+1}r_{t+1}f(z_t)|I_t) = 0 \tag{7}$$

where $f(z_t)$ is a square-integrable function of a possibly vector-valued $z_t \in I_t$. By the law of iterated expectations, this is the case unconditionally as well as conditionally:

$$E(E(m_{t+1}r_{t+1}f(z_t)|I_t)) = E(m_{t+1}r_{t+1}f(z_t)) = 0. \tag{8}$$

That is, the kernel must price both conditionally and unconditionally the asset excess-return and, therefore, it must price the payoffs, $r_{t+1}f(z_t)$, on all trading strategies that exploit conditioning information useful to predict it. In particular, we can set $f(z_t) = \mu(z_t) - \bar{\mu}$ and recognize $s_{t+1} := r_{t+1}(\mu(z_t) - \bar{\mu})$ as the excess-return on *a strategy that exploits the asset excess-return predictability captured by the predictive model in* (2). Then, since $E(m_{t+1}s_{t+1}) = Cov(m_{t+1}, s_{t+1}) + E(m_{t+1})E(s_{t+1})$, (8) implies

$$Cov(m_{t+1}, s_{t+1}) + E(m_{t+1})E(s_{t+1}) = 0. \tag{9}$$

Solving for $E(s_{t+1})$, (9) implies $E(s_{t+1}) = -Cov(s_{t+1}, m_{t+1})/E(m_{t+1})$. Therefore, normalizing the kernel so that[4] $E(m_{t+1}) = 1$ and squaring both sides, we have

$$(E(s_{t+1}))^2 = (Cov(s_{t+1}, m_{t+1}))^2. \tag{10}$$

Then, dividing both sides of the equality in (10) by $\sigma(s_{t+1})^2$, we have $\frac{(E(s_{t+1}))^2}{\sigma^2(s_{t+1})} = \frac{(Cov(s_{t+1},m_{t+1}))^2}{\sigma^2(s_{t+1})} = \rho^2(s_{t+1}, m_{t+1})\sigma^2(m_{t+1})$. Hence,

$$SR^2\,(s_{t+1}) = \rho^2(s_{t+1}, m_{t+1})\sigma^2(m_{t+1}). \tag{11}$$

When pricing excess-returns, the risk-free rate can be treated as if it were constant and known. In this case, as shown by Cochrane (1999), the squared *maximum* unconditional SR attainable by trading a given asset is the expectation of the asset squared conditional SR. The squared unconditional Sharpe ratio attainable by exploiting the predictability of r_{t+1} is, therefore,

$$SR^2\,(s_{t+1}) = E\left(SR^2_t\,(s_{t+1})\right). \tag{12}$$

The conditional SR of s_{t+1} is generated by a single asset, i.e. the asset with excess-return r_{t+1}. Hence, $SR_t\,(s_{t+1}) = SR_t\,(r_{t+1})$ and, therefore, (12) implies that

$$SR^2\,(s_{t+1}) = E\left(SR^2_t\,(r_{t+1})\right). \tag{13}$$

Using (13), we can then rewrite (6) as follows:

$$R^2 \leq SR^2\,(s_{t+1})\left(1-R^2\right) - SR^2(r_{t+1}). \tag{14}$$

Therefore, since $0 \leq R^2 < 1$ and $SR^2(r_{t+1}) \geq 0$, we have that

$$R^2 \leq SR^2\,(s_{t+1}). \tag{15}$$

Then, as required, (11) and (15) imply

$$R^2 \leq \phi \equiv \rho^2(s_{t+1}, m_{t+1})\sigma^2(m_{t+1}). \quad \square \tag{16}$$

The bound in (16) is tightest when m_{t+1} is the least volatile of the pricing kernels in the economy. For a given choice of m_{t+1}, it is closely related to the bound proposed by Huang and Zhou (2017) and given in equation (A-8) of their Appendix, which is

$$R^2 \leq \phi^2_{x,rz}\sigma^2(m_{t+1}), \tag{17}$$

where

$$\phi^2_{x,rz} \equiv \rho^2_{x,rz}\frac{\sigma^2(r_{t+1}(z_t - \mu_z))}{\sigma^2(r_{t+1})\sigma^2(z_t)}. \tag{18}$$

Here, $\rho^2_{x,rz}$ is the multiple correlation coefficient between x_{t+1} and $r_{t+1}(z_t - \mu_z)$, where $\mu_z \equiv E(z_t)$ and $z_t \in I_t$, and x_{t+1} is a vector of state variables[5] of the pricing kernel, so that $m_{t+1} = f(x_{t+1})$ for some measurable function f. For example, in the CAPM, $f(x_{t+1}) = \alpha + \beta x_{t+1}$ and $x_{t+1} = r_{M,t+1}$, with $\alpha, \beta \in \mathbb{R}$, $\alpha > 0$ and $\beta < 0$. As shown by Huang and Zhou (2017), this bound holds under either one of two alternative assumptions. These are either that returns are normally distributed or that $E(\varepsilon_{t+1}|x_{t+1}) = 0$, where ε_{t+1} is the residual in the orthogonal decomposition $r_{t+1}(z_t - \mu(z_t)) = a + bx_{t+1} + \varepsilon_{t+1}$.

Their bound differs from ours because, in place of $\rho(s_{t+1}, m_{t+1})$, they use the quantity $\phi^2_{x,rz}$. This quantity is a product of two terms,

[3] This defines what, less formally, is often referred to as m_{t+1}"conditionally" pricing the excess-return on dynamic strategies on the asset.

[4] Such normalization is possible because we are working with excess-returns, since pricing of excess-returns does not identify the mean of the kernel.

[5] The state variables are commonly referred to as risk factors.