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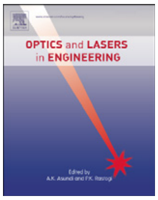

3D particle positioning by using the Fraunhofer criterion

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ABSTRACT

In particle tracking velocimetry, the necessary information is the 3D location of a given particle in space. This information can be obtained by examining the real image or by analyzing the interference fringe recorded on a digital camera. In this work, we measure the three-dimensional position of spherical particles by calculating the Central Spot Size of the interference pattern of a particle diffraction image. The Central Spot Size is obtained by combining the Continuous Wavelet transform and circle Hough transform. The Continuous Wavelet transform allow us in only one step enhanced quality of particle images and sets a threshold to select properly places where a Central Spot Size appear in order to determine its size via the Hough transform. The size and centroid of the Central Spot Size render z and x–y position of a particle image, respectively. The Central Spot Size is related to a criterion of a simplified theory given by the Fraunhofer theory in order to obtain z particle position. Our approach has been applied to simulated and experimental particle images. Simulated particle images show good agreement between actual and calculated Central Spot Size. An average relative error of 0.5% and 1.12% for x–y and z directions, respectively, was found in the analysis. Our experimental particle images were obtained from particle motion inside a channel. The quality of the particle images determines the accuracy of the calculation of the Central Spot Size of a particle image.

1. Introduction

Nowadays optical techniques for 3D particle positioning include scanning techniques [1], stereoscopic imaging [2], holographic methods [3–10] and quantitative defocusing methods [11–30]. These methods can be applied for the analysis of macro- or micro-fluids applications depending on the experimental arrangement utilized to fulfill such purpose.

Several methods based on defocusing methods have been developed to determine the out of plane position of spherical particles embedded in a fluid flow. Frequently, these methods are not based on correlation techniques but in single particle detection [3]. Single particle detection is a valuable tool in different subjects, such as colloidal sciences [9]. Some of these methods rely on intensity measurements from an experimental particle image to be compared against numerical calculations, where the particle light scattering is modeled using the classical the Near Field Lorenz–Mie Theory (NFLMT) or the Generalized Lorenz–Mie Theory (GLMT) [9,11–15]. However, light brightness non-uniformities are present in experimental particle images and make the defocus particle extraction difficult. In Refs. [9] and [15] were used the NFLMT and GLMT to extract particle position, and particle position and particle refractive index, respectively. However, the procedure to extract information is based in an optimization algorithm between the experimental and calculated intensity particle image, which makes the procedure a time consuming task

There are other approximations where the particle position is determined by using complicated experimental arrangements, and the numerical model to calculate the particle position neglects physical parameters of the particle such as light illumination, particle and medium refractive index [15–27]. Other authors, for its simplicity have used in-line digital holography to determine particle position [3,7–10]. In this technique, the extraction of particle position is performed by optical reconstruction. Also, hologram digital reconstruction was used to meet such a task, which makes the analysis simpler. Although, several process steps are needed to obtain the particle position; i.e. filtering, digital correlation and scanning different planes.

In this manuscript, we report on the use of the lensless in-line holography set-up to determine 3D particle position. However, in this study, we avoid the procedure proposed by other authors to calculate particle position [3–7]. Our approach is based on the determination of the Central Spot Size (*CSS*) of a particle diffraction image. It is well known that by measuring the *CSS* of the diffraction pattern of a particle image, the “z” (hereafter “z” and

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defocus particle position are used indistinctly) position of a particle can be determined [15,29,30]. The *CSS* is considered as the first ring of the diffraction pattern of a simulated or experimental particle image, such as the one seen in Fig. 1. The measured *CSS* can be related to an experimental calibration curve or to a numerical model predicted by the NFLM or GLMT in order to obtain the out of plane particle position [29,30].

In the present work, we relate the *CSS* to an expression obtained from the diffraction pattern of a spherical obstacle predicted by the Fraunhofer diffraction theory. By doing this approximation, we avoid optimization procedures and calibration curve calculations in order to determine 3D particle position. Also, knowledge of this criterion is important since using simplified theories enables us to handle less complicated formulas to save time in numerical computations.

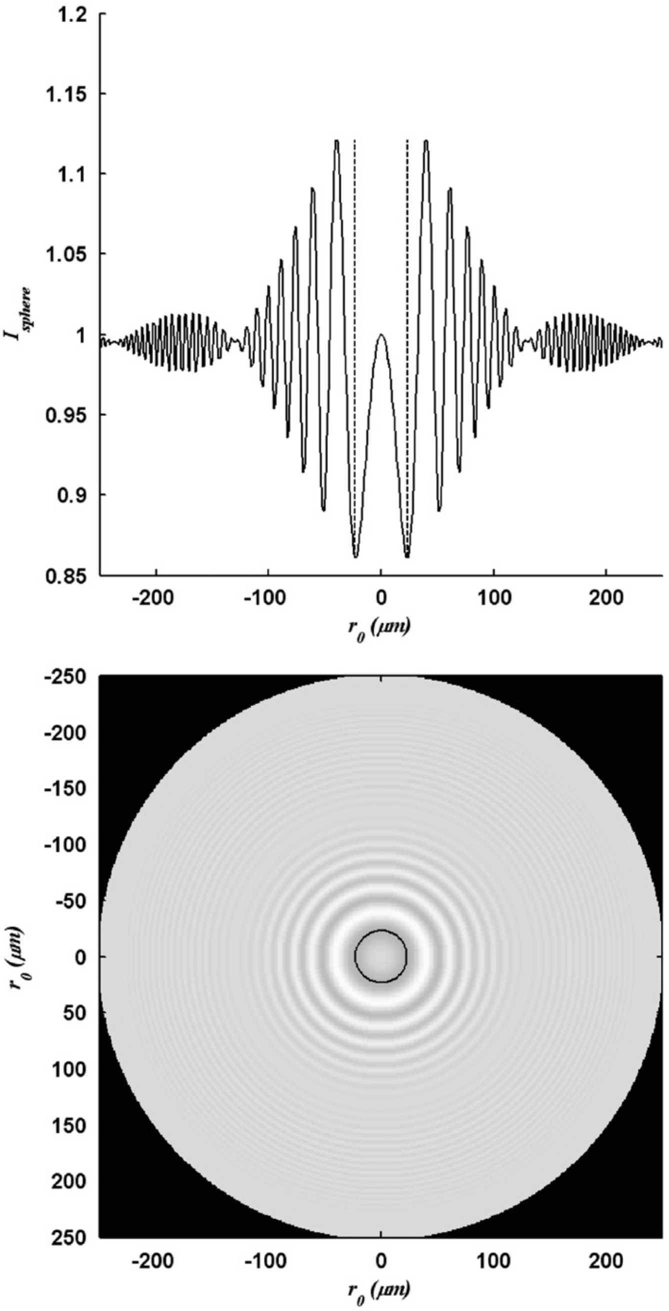

Fig. 1. Central Spot Size of a diffraction particle image. The simulated particle image was obtained using the Fraunhofer diffraction theory for a 10 μm diameter particle.

The *CCS* is determined from the diffraction pattern of the spherical particle by combining the Continuous Wavelet transform (CWT) and the Circle Hough transform (CHT) [31]. The function of each transform play a role in the procedure; i.e. the CWT allows us in only one step to locate and enhance the quality of particle images, and the CHT is used to determine the size of the circle of the *CSS* [31].

Our approach has been applied to simulated particle images and preliminary experimental results. The simulated particle images have been obtained by using the Fraunhofer diffraction theory for particles of 10 μm of diameter. The experimental particle images used in this work come from the lensless in-line holography set-up. In the experiment, we used a home-made glass channel filled with distilled water seeded with polystyrene particles of 10 μm of diameter.

2. Theoretical background

2.1. Diffraction theory

When calculating the diffraction pattern from a spherical object, one first calculates the diffraction pattern from an aperture of the same diameter. After the diffraction pattern from the aperture has been determined, the field diffracted by the object can be calculated by using Babinet's principle.

Consider a plane wave illuminating a circular aperture, as in Fig. 2, the electromagnetic field U at any point in space can be calculated using the Fresnel–Kirchhoff diffraction integral [32],

$$U(p)=\frac{1}{i\lambda}\iint_{aper}\frac{e^{ikR}}{R}B\,dA \tag{1}$$

where $k=2\pi/\lambda$, B is the obliquity factor (which is often set to unity as an approximation), λ is the wavelength of the light, $i=\sqrt{-1}$ and R is defined in Fig. 2. Eq. (1) is evaluated in cylindrical coordinates and by using the first two terms of the Taylor series

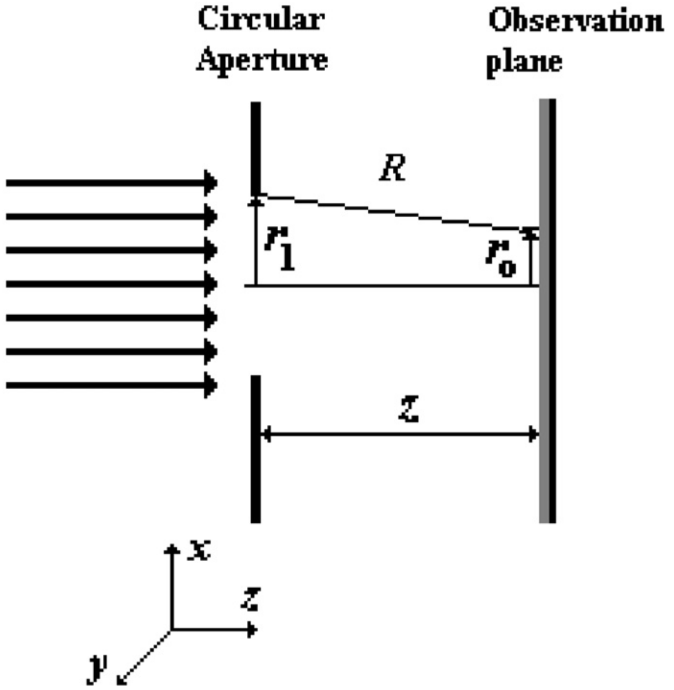

Fig. 2. Diffraction theory geometry for a circular aperture.