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On testing for structural break of coefficients in factor-augmented regression models☆

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HIGHLIGHTS

- This paper tests for structural break in factor-augmented regression models.
- We extend the classical structural break tests to this case by the two-step tests.
- Monte Carlo simulations show that the tests have good finite-sample performance.

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ABSTRACT

This paper considers testing for structural break of factor-augmented regression models with unknown change point. In this case, the classical structural break tests proposed by Andrews (1993) and Andrews and Ploberger (1994) are infeasible due to the presence of unobservable factors. This paper develops the feasible two-step tests based on their structural break tests. We prove that the asymptotic null distributions of the proposed two-step tests remain to be the same as those of their infeasible tests. The Monte Carlo simulations confirm the theoretical results and show that the two-step tests perform well in finite sample.

1. Introduction

Empirical researchers have found that variations in a large number of macroeconomic time series can be well explained by a few unobservable factors. Considering this, Stock and Watson (2002) and Bai and Ng (2006) incorporate the factors into an otherwise standard regression model and then propose factor-augmented regression model. The model assumes that the regression coefficients are time invariant. However, there is strong evidence that a subset of macroeconomic time series has undergone structural breaks, which may imply breaks in coefficients (see Stock and Watson, 1996).

It is well known that failure to take structural breaks into account may lead to invalid inference and incorrect policy implications, hence it is important to test for structural breaks. Recently the literature investigates tests for structural breaks in the factor loadings, including, Breitung and Eickmeier (2011), Baltagi et al. (2017), Bates et al. (2013), Chen et al. (2014), Cheng et al. (2016), and Han and Inoue (2015), to name a few. Baltagi et al. (2016), Li et al. (2016) and Wang et al. (2015) focus on the estimation of factor-augmented regression models given the presence of structural breaks. The purpose of this paper is to test for one-time break of the coefficients of observable regressors in factor-augmented regressions. To the best of our knowledge, there is one test for structural instability of factor augmented regression models proposed by Corradi and Swanson (2014) (henceforth CS), who develop a test based on the difference between a full sample and a rolling window estimator of the covariance of the explained variable and the estimated factors. Considering of the finite sample performance, they recommend to inference using bootstrap critical values obtained from block bootstrap (henceforth CS_B). The implementation of their test is simple but their test is inconsistent under certain cases.

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In standard regression models, the classical tests for structural change with unknown change point are typically attributed to Andrews (1993) and Andrews and Ploberger (1994). They propose the supreme Wald test statistics (supW), the mean Wald statistics (meanW) and the exponential Wald statistics (expW) respectively. However, for factor-augmented regressions, the above three tests are not feasible due to the presence of unobservable factors. This paper extends the above tests to the case of factor-augmented regression models by a two-step procedure: first, we estimate the factors by the principal components analysis (PCA); second, we treat the estimated factors as the underlying factors and use supW, meanW and expW to test for structural break. We will show that the above tests have the same asymptotic null distributions as if the factors were observed, thus one can conduct the proposed tests using the critical values from Andrews (1993) and Andrews and Ploberger (1994). Given the wide applicabilities of the above infeasible tests and factor-augmented regression models, the feasible tests proposed in this paper can be of considerable practical importance.

The rest of the paper is organized as follows. In Section 2, we present the basic notations and introduce the test statistics. Section 3 establishes the asymptotic properties of the suggested test statistics under the null hypothesis. In Section 4, we investigate the finite sample performance of the tests. Section 5 concludes.

2. Notations and test statistics

Consider the following factor-augmented regression model with a single structural break:

$$y_t = x_t'\mathbf{1}(1 \le t \le \pi_0 T)\beta_1 + x_t'\mathbf{1}(\pi_0 T < t \le T)\beta_2 + f_t'\gamma + v_t, \quad (1)$$

where x_t is a vector of p observable regressors which can contain intercept and v_t is a real-valued idiosyncratic error for $t = 1, 2, \ldots, T$. $\mathbf{1}(\cdot)$ is an indicator function, $\pi_0 \in (0, 1)$ is the potential break fraction. For simplicity, πT denotes $\lfloor \pi T \rfloor$, where $\lfloor \cdot \rfloor$ is the integer part operator. The unobservable regressors f_t is an r-vector of factors which can be arbitrarily correlated to x_t. Factors represent diffusion indices that measure the effects of systematic shocks or risks. Then we assume that factors come from the approximate factor models of w_{it} with factor loading λ_i as follows

$$w_{it} = \lambda_i' f_t + e_{it}, \quad i = 1, 2, \ldots, N. \quad (2)$$

With model (1) and (2), we intend to test the following hypotheses:

$$H_0 : \beta_1 = \beta_2, \quad (3)$$

versus

$$H_A : \beta_1 \ne \beta_2. \quad (4)$$

To propose the test statistics, we need to introduce some matrix notations. Stacking the observations of y_t, v_t, w_{it} and e_{it} over t, we have four $T \times 1$ vectors y, v, w_i and e_i. For the factor model, define $W = (w_1, \ldots, w_N)$, $F = (f_1, \ldots, f_T)'$, $\Lambda = (\lambda_1, \ldots, \lambda_N)'$ and $e = (e_1, \ldots, e_N)$. For the ease of exposition of structural break, let $X_1 = (x_1, \ldots, x_{\pi T}, 0, \ldots, 0)'$, $X_2 = (0, \ldots, 0, x_{\pi T+1}, \ldots, x_T)'$. If we replace π by π_0 in X_1 and X_2, we have X_1^0 and X_2^0. Then we define $Z = (X_1, X_2, F)$ and $Z^0 = (X_1^0, X_2^0, F)$. With these notations, the model can be reformulated into a matrix form,

$$y = X_1^0\beta_1 + X_2^0\beta_2 + F\gamma + v = Z^0\delta + v, \quad W = F\Lambda' + e$$

where $\delta = (\beta_1', \beta_2', \gamma')'$. If f_t were observed, we can test the null hypothesis (3) as in Andrews (1993) and Andrews and Ploberger (1994) by the supW, meanW and expW test statistics. We denote them as supW(F), expW(F) and meanW(F), respectively.

However, the factors f_t are unobservable, thus the above tests are infeasible. Following Wang et al. (2015), we use a two-step procedure to estimate the model: In the first step, we use principal component method to estimate F by $\hat{F} = (\hat{f}_1, \ldots, \hat{f}_T)'$, which is the $T \times r$ matrix consisting of r eigenvectors (multiplied by $\sqrt{T}$) associated with the r largest eigenvalues of the matrix $WW'/(TN)$ in decreasing order. Next, treating the estimated factors $\hat{F}$ as the underlying factors F, we regress y on $\hat{Z} = (X_1, X_2, \hat{F})$ and then get the ordinary least squares estimator (OLS) of δ, denoted by $\hat{\delta} = (\hat{\beta}_1', \hat{\beta}_2', \hat{\gamma}')$. Let $\hat{z}_t$ be the transpose of the tth row of $\hat{Z}$. Then the corresponding residual will be $\hat{v}_t = y_t - \hat{\delta}\hat{z}_t$. Following Andrews (1993), the Wald test statistic is then defined as follows

$$\mathrm{W}(\pi, \hat{F}) = T\hat{\delta}'R'\left(R\left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}\hat{\Omega}_T\left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}R'\right)^{-1}R\hat{\delta} \quad (5)$$

where $R = (I_p, -I_p, 0_{p\times r})$, $\hat{\Omega}_T$ is the heteroscedasticity and autocorrelation consistent (HAC) estimate of the long run variance of $\{\hat{z}_t\hat{v}_t\}$, which will be defined more precisely in the next section. Correspondingly, the supW, meanW and expW test statistics can be constructed as:

$$\mathrm{supW}(\hat{F}) = \sup_{\pi\in\Pi}\mathrm{W}(\pi, \hat{F}),$$

$$\mathrm{meanW}(\hat{F}) = \ln\left(\frac{1}{\pi_2 - \pi_1}\int_{\pi_1}^{\pi_2}\exp\left(\mathrm{W}(\pi, \hat{F})/2\right)d\pi\right),$$

$$\mathrm{expW}(\hat{F}) = \frac{1}{\pi_2 - \pi_1}\int_{\pi_1}^{\pi_2}\mathrm{W}(\pi, \hat{F})d\pi,$$

where $\Pi = [\pi_1, \pi_2]$ is any set whose closure lies in $(0, 1)$.

Note that using true factors F instead of estimated factors $\hat{F}$, we follow the approach of construction of the Wald test statistics Eq. (5), we can obtain the infeasible Wald statistic $\mathrm{W}(\pi, F)$. Then $\mathrm{supW}(F) = \sup_{\pi\in\Pi} W(\pi, F)$, $\mathrm{meanW}(F) = \ln\left(\frac{1}{\pi_2-\pi_1}\int_{\pi_1}^{\pi_2}\exp\left(\frac{W(\pi,F)}{2}\right)d\pi\right)$ and $\mathrm{expW}(F) = \frac{1}{\pi_2-\pi_1}\int_{\pi_1}^{\pi_2} W(\pi, F)\, d\pi$.

3. Assumptions and inference theory

To analyze the asymptotic properties of the tests, we make the following assumptions. Hereafter, $\|\cdot\|$ denotes the Euclidean norm of a vector or matrix, $\Longrightarrow$ denotes weak convergence of stochastic processes, C is a generic positive constant large enough.

Assumption A (*Factors*). $E\|f_t\|^4 \le C$, $\frac{1}{T}\sum_{t=1}^{T} f_t f_t' \xrightarrow{p} \Sigma_f$ as $T \to \infty$ for some non-random positive matrix Σ_f.

Assumption B (*Loadings*). $\|\lambda_i\| \le C$, $\frac{1}{N}\Lambda'\Lambda \xrightarrow{p} \Sigma_\Lambda > 0$.

Assumption C (*Time and Cross-Section Dependence with Heteroscedasticity*). Let $E(e_{is}e_{jt}) = \sigma_{ij,st}$, we assume that

1. $E(e_{it}) = 0$ and $E|e_{it}|^8 \le C$.
2. $\frac{1}{NT}\sum_{i=1}^{N}\sum_{i=j}^{N}\sum_{s=1}^{T}\sum_{t=1}^{T}|\sigma_{ij,st}| \le C$.
   There exist $|\sigma_{ij,tt}| \le \bar{\sigma}_{ij}$ for all t such that $\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{\sigma}_{ij} \le C$.
   There exists $\gamma_{st} = E(\frac{1}{N}\sum_{i=1}^{N}\sigma_{ii,st})$ such that $\sum_{s=1}^{T}|\gamma_{st}| \le C$.
3. For every (t, s), $E\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}[e_{is}e_{it} - E(e_{is}e_{it})]\right)^4 \le C$.

Assumption D. There exists an $C < \infty$ such that

1. $E\left(\frac{1}{N}\sum_{i=1}^{N}\|\frac{1}{\sqrt{T}}\sum_{t=1}^{T}f_t e_{it}\|^2\right) \le C$.
2. $E\|\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}f_s[e_{is}e_{it} - E(e_{is}e_{it})]\|^2 \le C$.
3. $E\|\frac{1}{\sqrt{NT}}\sum_{t=1}^{T}\sum_{i=1}^{N}f_t\lambda_i'e_{it}\|^2 \le C$.
4. For each t, $E\left\|\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\lambda_i e_{it}\right\|^4 \le C$.