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The suppression of phase error by applying window functions to digital holography

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ABSTRACT

Digital holography (DH) is a 3D imaging technique with a theoretical axial accuracy of around 1–2 nm. However, in practice, the axial error is generally quoted as tens of nanometers. Previous studies on sources of axial error mainly focused on the phase error introduced by lens. However, it was later shown that other factors such as the limited CCD aperture size also contribute to axial error. Based on this study, further investigation approaches to suppress the axial error caused by the limited CCD aperture size is discussed in this paper. Use of a window function to modify the shape of the hologram aperture after the recording process is proposed to reduce the axial error. The mechanism of how this window function reduces axial/phase error is analyzed. Specific features of this window function related to the axial error, namely the side lobe energy to main lobe energy ratio (SMER), is postulated. Both simulation and experiment are performed to validate that the selection of an appropriate window function helps to reduce the axial error of digital holography and SMER is an effective indicator in selection of an appropriate window function.

1. Introduction

Digital holography (DH) [1–6], compared to conventional holography, has many advantages including the access to quantitative amplitude and phase information. The quantitative phase information contains depth information of the object and thus allows 3D imaging of the object. DH has been widely applied in 3D microscopic measurements in life and material sciences [7–14].

One of the advantages of DH over other 3D measurement and imaging techniques is its excellent theoretical axial measurement accuracy of better than 1 nm limited by the quantization effect of CCD pixel [15]. However, in practice, the axial error is generally of tens of nanometers, much higher than the theoretical value [16–20] . This implies there are other factors limiting the axial accuracy of DH. Therefore it is necessary to identify other axial error sources and investigate the corresponding approaches to reduce these errors.

The axial error of DH system have been studied earlier [21–26]. However, these studies mainly focused on the phase error introduced by lens, such as phase aberrations. Recently, it was reported that CCD aperture size is also an important contributor to axial error [27,28]. Larger CCD size results in smaller axial error. Following from these researches, further axial error reduction approaches are proposed and demonstrated in this paper.

Increasing the CCD size and hence the hologram size is not a practical solution from both set-up and speed of measurement aspects. However, it is possible to modify the aperture shape of the digital hologram by applying a window function to the recorded digital hologram. Window functions had been applied to DH for diffraction suppression [29,30], in order to improve the lateral resolution of DH image. However, their impacts on the phase image and the associated axial measurement error have not been studied. In this work, it is proposed and demonstrated that the axial error can be successfully suppressed by applying appropriate window functions. It is postulated that a specific feature, the side lobe energy to main lobe energy ratio (SMER), is directly related to the axial error. By applying a window function with an appropriate SMER value, the axial error is effectively reduced as demonstrated through simulation and experiment. The results verify the effectiveness of window functions in the axial error reduction and the effectiveness of SMER indicator in the selection of appropriate window functions for such purpose.

This paper is organized as follows: Section 2 introduces the

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theory model of the application of window functions to hologram. In Section 3, simulations of the application of window functions are performed. The impacts of the application on the axial error are analyzed. In Section 4, experiments are performed and the experimental results are discussed. Final conclusion is given in Section 5.

2. Theoretical model

In this section, we discuss the theoretical basis of the application of window function to a recorded hologram and derive the corresponding mathematical model to facilitate the analysis.

DH includes two major processes: digital recording and numerical reconstruction [22]. The hologram is the interference pattern of the object wave o and the reference wave r at the CCD recording plane and ban be mathematically [22] expressed as

$$h = |r + o|^2 = rr^* + oo^* + ro^* + or^*, \tag{1}$$

The application of window function w to the hologram h is described mathematically as

$$h_w = h \times w = rr^*w + oo^*w + ro^*w + or^*w, \tag{2}$$

where $\times$ and $*$ are the multiplication and conjugate operators respectively. h_w is the window function modified hologram.

Without loss of generality and for ease of processing, an off-axis DH set up is considered. For off-axis configurations, the numerical reconstruction of the object image is based on the spatial filtering of the first three terms of Eqs. (1) and (2) in the Fourier domain of the hologram. Then, the fourth term is left and multiplied with the reference wave r. In the case without the application of window function, the term $|r|^2 o$ is recovered and produces a real image of the object. In the case with the application of window function, the term $|r|^2 ow$ is obtained and produces a window function modified real image of the object. In both cases, the factor $|r|^2$ only influences the brightness of the image. It does not affect the phase of the image. The utilization of the reference wave in off-axis DH is just to assist the acquisition of the digital complex object wave o. Its impact is not the focus of this work and hence will not be considered in this paper. We directly focus on the complex object wavefield ow instead.

The whole process of DH including the application of window function is described in Fig. 1 (without the consideration of the reference wave r). Due to the separable property of Fresnel transform, only one dimension case is considered and it can be extended to two-dimension easily. First, the object wave $s(x)$ at the object plane propagates from the object to the CCD plane and becomes the object wave at the CCD plane $o(x)$ which interferes with the reference beam at the CCD. After the digital recording of the hologram, a window function $w(x)$ is applied to the hologram. In the numerical reconstruction process, the window function modified object wavefront $m(x) = w(x)o(x)$ is recovered. $m(x)$ is further numerically back propagated to image plane to reconstruct the image $i(x)$ in numerical reconstruction process. The whole process can be expressed mathematically as Eq. (3) in spatial domain:

$$i(x) = Fresnel^{-1}\left\{ Fresnel\left\{ s(x)\right\} \times w(x)\right\}, \tag{3}$$

where *Fresnel* and *Fresnel*$^{-1}$ denotes Fresnel diffraction integral and inverse Fresnel diffraction integral respectively. The Fresnel diffraction integral expresses the free-space propagation process of light in the near field of object [31]. It expresses the relation between the object wave $s(x)$ at the object plane and the object wave $o(x)$ at the CCD plane. The object wave at the CCD plane $o(x)$ is the convolution of the object $s(x)$ and the Fresnel diffraction impulse response which is the amplitude point spread function (APSF) of the free-space propagation [31]. The convolution in spatial domain is replaced by product in frequency domain. The Fourier transform of the APSF is named the amplitude or coherent transfer function of propagation through free space as $H(f)$ [31]. According to the above knowledge, we extend the Fresnel diffraction integral in Eq. (3) and it is expressed by Eq. (4) as below:

$$i(x) = Fourier^{-1}\left\{ \left[(S(f) \times H(f)) \otimes W(f)\right] \times H^*(f)\right\}, \tag{4}$$

where $S(f)$ and $W(f)$ are the Fourier transforms of $s(x)$ and $w(x)$. $\otimes$is convolution operator. *Fourier*$^{-1}$ and $H(f)$ denotes the inverse Fourier transform, coherent transfer function of propagation through free space, respectively. Simplifying and substituting $H(f) = e^{jkz}e^{-j\pi\lambda z f^2}$ [31] into Eq. (4) gives

$$\begin{aligned} i(x) &= Fourier^{-1}\left\{ \int_{\hat f} S(\hat f)H(\hat f)W(f-\hat f)d\hat f \times H^*(f)\right\} \\ &= \int_f \int_{\hat f} S(\hat f)e^{jkz}e^{-j\pi\lambda z\hat f^2}W(f-\hat f)d\hat f\, e^{-jkz}e^{j\pi\lambda z f^2}e^{j2\pi xf}df \\ &= \int_f \int_{\hat f} W(f-\hat f)S(\hat f)e^{-j\pi\lambda z\left(\hat f^2 - f^2\right)}e^{j2\pi xf}d\hat f\, df, \end{aligned} \tag{5}$$

where f and $\hat f$ represent frequency.

Eq. (5) now permits the study of the impact of different windows functions on the axial error.

3. Simulation investigation and analysis

In this section, effect of different window functions on the axial error is studied through simulation. This study would help identify the specific factor of window function which contributes to the axial error and could be used as the indicator for window function selection to suppress the axial error.

3.1. Simulation of the application of window functions

In simulation, Hann, Hamming, Taylor and rectangle windows

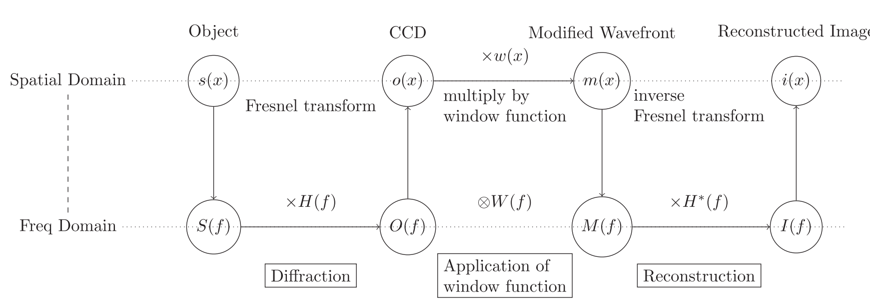

Fig. 1. The diffraction of object wavefield and reconstruction.