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# Error-compensating phase-shifting algorithm for surface shape measurement of transparent plate using wavelength-tuning Fizeau interferometer

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ABSTRACT

When measuring the surface shape of a transparent sample using wavelength-tuning Fizeau interferometry, the calculated phase is critically determined by not only phase-shift errors, but also by coupling errors between higher harmonics and phase-shift errors. This paper presents the derivation of a *13*-sample phase-shifting algorithm that can compensate for miscalibration and first-order nonlinearity of phase shift, coupling errors, and bias modulation of the intensity, and has strong suppression of the second reflective harmonic effect. The characteristics of the *13*-sample algorithm are estimated with respect to Fourier representation in the frequency domain. The phase error of measurement performed using the *13*-sample algorithm is discussed and compared with those of measurements obtained using other conventional phase-shifting algorithms. Finally, the surface shape of a fused silica wedge plate obtained using a wavelength tuning Fizeau interferometer and the *13*-sample algorithm are presented. The experimental results indicate that the surface shape measurement accuracy for a transparent fused silica plate is 3 nm. The accuracy of the measurement is discussed by comparing the amplitudes of the crosstalk noise calculated using other conventional algorithms.



## 1. Introduction

Surface shape is a fundamental characteristic of transparent optical devices used in precision measurements and the semiconductor industry. Atomic force microscopes (AFMs) have been widely used in the semiconductor industry to measure the surface shape of optical devices precisely. However, measurements using AFMs require a considerable amount of time to measure the entire surface shape distribution [1,2].

Surface shape measurement of a large-diameter transparent sample can be achieved using wavelength-tuning Fizeau interferometry. In wavelength-tuning interferometry, the phase difference between a sample beam and a reference beam is varied by phase shifting, and the signal irradiance is acquired at equal intervals of the phase difference [3]. The phase distribution of a fringe pattern can be calculated using a phase-shifting algorithm. Recently, common-path interferometers [4,5] have also been used for surface profiling when detecting wide defects on a silicon wafer [6].

Phase-shift errors and nonsinusoidal waveforms of the signal are the most common sources of systematic errors in phase evaluation [7]. Moreover, the effects of the coupling errors between phase-shift errors and higher harmonic components should be considered [8]. In particular, the effects of second harmonic components become significantly large when measuring a transparent surface shape or a highly reflective surface shape [8,9], because the reflectivities of the reference and sample surfaces should be considered. The effect of multireflection can be suppressed by using other methods, e.g., broadband light sources, such as a mode-locked laser [10] and white light [11], and antireflection coatings on the reference surface [12,13]. However, the measurement methods using a broadband light source give rise to low fringe contrast, high random noise, and poor signal-to-noise ratio. Moreover, antireflection coatings on the reference surface deteriorate the accuracy of the surface. Hence, coatings influence the total uncertainty of surface shape measurement. The photon shot noise of a charged-coupled device (CCD) camera also affects measurement accuracy and should be considered when measuring surface shape using a wavelength-tuning Fizeau interferometer. Recently, a theory for determining phase errors

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based on the Poisson statistics of photons has been reported [14]. Using this theory, the phase errors resulting from photon shot noise can be formulated and estimated more statistically. When using a wavelength-tuning diode laser as the phase shifter, the nonlinearity of phase shift and intensity modulation during wavelength tuning degrade the measurement accuracy [15–17].

Several error-compensating phase-shifting algorithms minimize systematic phase errors [9,18–31]. The prominent Schwider–Hariharan five-sample algorithm [9,18] can compensate for phase-shift miscalibration but not the coupling errors between harmonics and phase-shift errors. Larkin and Oreb derived an $(N+1)$-sample symmetrical phase-shifting algorithm [19] based on the Fourier representation [32] and discussed the effects of nonsinusoidal waveforms and residual phase-shift errors. Schmit and Creath developed five-sample and six-sample algorithms [22] based on the extended averaging method. Using a data-sampling window, de Groot developed a seven-sample algorithm [23] that can compensate for up to second-order nonlinearity of phase shift. However, these algorithms do not compensate for coupling errors and intensity modulation during phase shifting. Surrel developed a windowed phase-shifting algorithm that can compensate for phase-shift miscalibration and coupling errors using the characteristic polynomial theory [24]. However, the windowed phase-shifting algorithm does not compensate for first-order nonlinearity of phase shift. Onodera derived a six-sample algorithm that is insensitive to intensity modulation [25], and Surrel described this insensitivity using the characteristic polynomial theory [26]. However, Onodera's six-sample algorithm cannot suppress second harmonic components or be applied to the surface measurement of transparent plates.

We have already developed a $4N-3$ algorithm [31] that can compensate for up to second-order nonlinearity of phase shift and coupling errors. However, the $4N-3$ algorithm is difficult to utilize for actual measurements in the manufacturing industry. In Ref. [31], 61 images were acquired for measurement, though it is difficult to acquire so many images in the glass manufacturing industry. In actual interferometric measurements, it is preferable to use as few images as possible to reduce time and technical problems.

This paper presents the derivation of a *13*-sample phase-shifting algorithm that can compensate for phase-shift miscalibration, first-order nonlinearity in phase shift, coupling errors, and bias modulation of the intensity, and has a strong suppression ability for second harmonic components. It is shown that the *13*-sample algorithm yields the smallest phase errors compared with conventional phase-shifting algorithms. Finally, the surface shape of a transparent fused silica wedge plate measured using a wavelength-tuning Fizeau interferometer and the *13*-sample algorithm is presented. The measurement accuracy is discussed by comparing the amplitudes of crosstalk noise calculated using other phase-shifting algorithms.

## 2. 13-Sample phase-shifting algorithm

### 2.1. Phase-shifting algorithm and characteristic polynomial theory

A laser Fizeau interferometer (Fig. 1) allows the interference of multiple reflections between a sample surface and a reference surface by virtue of the high degree of coherence of the light. Let the reference and sample surface reflectivities be $r_1$ and $r_2$, respectively.

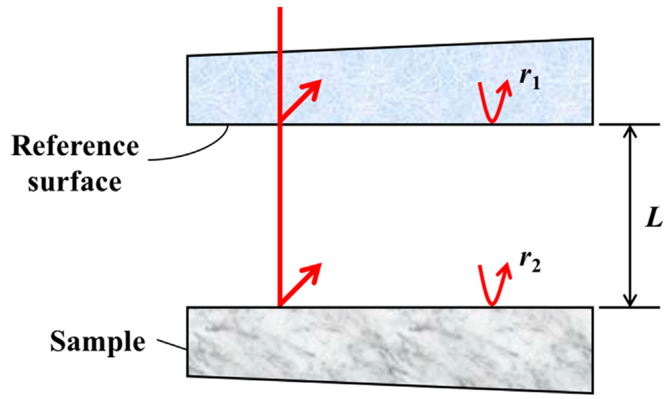


**Fig. 1.** Laser Fizeau interferometer.

From Fig. 1, the observed signal irradiance $I(\alpha_r)$ in the interference fringe pattern during phase shifting is given by [8,28]

$$I(\alpha_r) = \sum_{m=1}^{\infty} A_m \cos(\varphi_m - m\alpha_r) = I_0\left[1 + \sum_{m=1}^{\infty} \gamma_m \cos(\varphi_m - m\alpha_r)\right]$$
$$= I_0 + I_0\gamma_1 \cos(\varphi_1 - \alpha_r) + I_0\gamma_2 \cos(\varphi_2 - 2\alpha_r) + \cdots, \quad (1)$$

where $\alpha_r$ is the phase-shift parameter, and $A_m$ and $\varphi_m$ are the amplitude and phase of the $m$th harmonic component, respectively. The DC component $I_0$ of the signal irradiance and the fringe contrast $\gamma_m$ of the $m$th harmonic components are given by [8,28]

$$I_0 = \frac{r_1 + r_2 - 2r_1r_2}{1 - r_1r_2}, \quad (2)$$

$$\gamma_1 = \frac{2(1 - r_1)(1 - r_2)}{r_1 + r_2 - 2r_1r_2}\sqrt{r_1r_2}, \quad (3)$$

$$\gamma_2 = -\gamma_1\sqrt{r_1r_2}. \quad (4)$$

The $\gamma_m$ for successive harmonics of order $m$ decreases in strength by a factor of $-\sqrt{r_1r_2}$. The phase distribution $\varphi_1$ can be determined using a phase-shifting algorithm. A general expression for the calculated phase in $M$-sample algorithm is given by [7]

$$\varphi^* = \arctan\frac{\sum_{r=1}^{M} b_r I(\alpha_r)}{\sum_{r=1}^{M} a_r I(\alpha_r)}, \quad (5)$$

where $a_r$ and $b_r$ are the $r$th sampling amplitudes, and $I(\alpha_r)$ is given by Eq. (1). When the phase shift is nonlinear, each $\alpha_r$ value is a function of the phase-shift parameter. It can be expressed as a polynomial function of the unperturbed phase-shift value $\alpha_{0r}$ as [27]

$$\alpha_r = \alpha_{0r}\left[1 + \varepsilon(\alpha_{0r})\right]$$
$$= \alpha_{0r}\left[1 + \varepsilon_0 + \varepsilon_1\frac{\alpha_{0r}}{\pi} + \varepsilon_2\left(\frac{\alpha_{0r}}{\pi}\right)^2 + \cdots + \varepsilon_p\left(\frac{\alpha_{0r}}{\pi}\right)^p\right], \quad (6)$$

where $p$ is the maximum order of the nonlinearity, $\varepsilon_0$ is the error coefficient of the phase-shift miscalibration, $\varepsilon_q$ $(1 \le q \le p)$ is the error coefficient of the $q$th nonlinearity of the phase shift, and $\alpha_{0r}=2\pi[r-(M+1)/2]/N$ is the unperturbed phase shift.

The phase error $\Delta\varphi$ in the calculated phase is a function of the amplitude ratios $A_m/A_1$ and of the error coefficients $\varepsilon_q$, and it can be expanded as the following Taylor's series