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Three-frame generalized phase-shifting interferometry by a Euclidean matrix norm algorithm

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ABSTRACT

Generalized phase-shifting interferometry (GPSI) is one of the most effective techniques in imaging of a phase object, in which phase retrieval is an essential and important procedure. In this paper, a simple and rapid algorithm for retrieval of the unknown phase shifts in three-frame GPSI is proposed. Using this algorithm, the value of phase shift can be calculated by a determinate formula consisting of three different Euclidean matrix norms of the intensity difference between two phase shifted interferograms, and then the phase can be retrieved easily. The algorithm has the advantages of freeing from the background elimination and less computation, since it only needs three phase-shifted interferograms without no extra measurements, the iterative procedure or the integral transformation. The reliability and accuracy of this algorithm were demonstrated by simulation and experimental results.

1. Introduction

Phase-shifting interferometry (PSI) is an important technique widely used in optical measurement and microscopy [1–4]. In traditional PSI, a special constant phase shift, $2\pi/N$ with the integer $N \geq 3$ between two adjacent frames, is often assumed. Subsequently, Greivenkamp [5] and Stoilov [6] proposed general methods to deal with arbitrary phase shifts. Although the phase shifts do not have to be special values, they are either known precisely or equal in these methods. In fact, it is still a strict requirement on the precision of the phase shifter. In order to remove this requirement, many methods have been reported [7–18], among which the generalized phase shift exaction techniques [10–18] provide a possible method for extracting arbitrary and unknown phase shifts from holograms directly.

In general, the phase shift extraction methods can be classified into two categories: iterative and noniterative. The iterative methods [10,11] are greatly time-consuming since the procedures are repeated many times to achieve acceptable accuracy. For example, an advanced iterative algorithm based on the least squares method is proposed to determine the phase shift and the phase distribution simultaneously [10]. For this reason, noniterative methods [12–21] have been favored for their speed and less computational loads. Cai and other researchers proposed a class of statistical algorithms to calculate the unknown phase shift directly and reconstruct the wavefront from two or more holograms [12–15]. These algorithms are based on the assumption that the measured object has a random phase in $[0, 2\pi]$ over the whole interferogram. To relax this assumption, another method has been proposed to extract the phase shift accurately using the histogram of phase difference between two adjacent frames [16]. However, it requires determining both the background intensity and the modulation amplitude by searching for the maximum and the minimum intensity in each pixel of interferograms, which still takes considerable time. Later, a self-tuning approach is proposed to retrieve the phase shift by looking for the minimum of a merit function [17], where the accuracy of phase shift decreases when the phase shift is far from $\pi/2$ and the interferograms are required to be normalized beforehand. In Ref. [18], an accurate phase shift extraction algorithm is proposed by using the maximum and the minimum values of the interference term. In Ref. [19], the Gram-Schmidt (GS) orthonormalization algorithm is employed to extract the phase with high precision and rapid speed. Both of these two-step demodulation methods need a precondition of filtering out the background term by a high-pass filter in advance. In Refs. [20,21], a new kind of phase-shifting demodulation method based on the use of principal component analysis (PCA) algorithm is proposed. It is worth noting that this method does not require the extraction of the phase shift to retrieve the modulating phase. Although the PCA method is fast, it still requires the elimination of the background term by a temporal average in advance. However, the filtering and averaging algorithms do not work well for the background elimination when the interferograms are with rapid

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background variation [22]. New methods that are robust to background vibration have been also proposed [23–25]. In Ref. [25], a combination of the GS orthonormalization process and the two-dimensional continuous wavelet transform (2D-CWT) algorithm is used to analyze two-step arbitrarily phase-shifted interferograms. The method works well when the interferograms contain complex fringes, large fringe-frequency variations, noise or defect fringes. It is still time-consuming because the 2D-CWT algorithm refers to several times Fourier transforms.

In this paper, we propose a Euclidean matrix norm (EMN) algorithm to extract the unknown phase shifts from only three interferograms. After substituting the phase shifts to the three-step phase-shifting algorithm, the phase can be retrieved easily. This algorithm is faster since it does not use the iterative procedure or the integral transformation. Moreover, it is easy to implement without the background elimination or the measurements of other parameters excepting the interferograms in the entire retrieval process. The only requirement is the fringe limit condition, which is easy to meet in real cases.

2. Method

For the three-frame generalized phase-shifting interferometry, the distribution of intensity for each interferogram can be given in the following form:

$$I_{kmn} = a_{mn} + b_{mn}\cos[\varphi_{mn} + \delta_k], \quad (k = 1, 2, 3) \tag{1}$$

where m and n denote the pixel position of rows and columns of interferograms respectively. a_{mn}, b_{mn} and φ_{mn} represent the background intensity, the modulation amplitude and the measured phase, respectively. The phase shift related to the kth interferogram, δ_k, is usually assumed to be zero when $k=1$. With the measured intensities, the difference between the pth and qth interferograms can be expressed as

$$\Delta I_{pq} = I_{pmn} - I_{qmn} = 2b_{mn}\sin\left[\phi_{mn} + \frac{\delta_q + \delta_p}{2}\right]\sin\left[\frac{\delta_q - \delta_p}{2}\right] \quad (p, q = 1, 2, 3) \tag{2}$$

Here, we consider the Euclidean matrix norm (EMN) of the intensity difference. In general, for a matrix $T = [t_{mn}]$, with $M \times N$ order, its EMN is defined as $\|T\|_2 = [\sum_{m=1}^{M}\sum_{n=1}^{N}(t_{mn})^2]^{1/2}$, in which the sign $\|\|_2$ is the EMN operator. Therefore, the EMN of ΔI_{pq} can be expressed as

$$\begin{aligned} E_{pq} &= \left\|\Delta I_{pq}\right\|_2 \\ &= \left\{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[2b_{mn}\sin\left(\phi_{mn} + \frac{\delta_q + \delta_p}{2}\right)\sin\left(\frac{\delta_q - \delta_p}{2}\right)\right]^2\right\}^{1/2} \\ &= \sqrt{2}\left|\sin\left(\frac{\delta_q - \delta_p}{2}\right)\right| \\ &\quad \left\{\sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}^2\left[1 - \cos\left(2\phi_{mn} + \delta_q + \delta_p\right)\right]\right\}^{1/2} \end{aligned} \tag{3}$$

If the fringe number in each interferogram is more than one, then the measured phase varies more than 2π (rad) in the observed area. As a result, the following approximation can be applied,

$$\sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}^2 >> \sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}^2\cos\left(2\varphi_{mn} + \delta_q + \delta_p\right) \tag{4}$$

Thus, Eq. (3) can be simplified as

$$E_{pq} \approx \sqrt{2}\left|\sin\left(\frac{\delta_q - \delta_p}{2}\right)\right|\left\{\sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}^2\right\}^{1/2} = C \times \left|\sin\left(\frac{\delta_q - \delta_p}{2}\right)\right| \tag{5}$$

with $C = \sqrt{2}\cdot\{\sum_{m=1}^{M}\sum_{n=1}^{N}b_{mn}^2\}^{1/2}$. From Eq. (5), it is clear that E_{pq} is proportional to $\left|\sin[(\delta_q - \delta_p)/2]\right|$, and there are only three unknown quantities, namely δ_2, δ_3 and C. To determine these quantities, at least three equations are required. According to Eq. (5), there are three quantities as follows:

$$E_{12} = C \times \left|\sin(\delta_2/2)\right|, E_{13} = C \times \left|\sin(\delta_3/2)\right|, E_{23} = C \times \left|\sin\left[(\delta_3 - \delta_2)/2\right]\right| \tag{6}$$

In order to avoid the uncertainty of the sign, the phase shift δ is normally constrained within the range of $[0, \pi]$. Therefore, $\delta/2$ ranges from 0 to $\pi/2$, and the function of $\sin(\delta/2)$ is monotonically increasing and positive. So $|\sin(\delta_2/2)| = \sin(\delta_2/2)$ and $|\sin(\delta_3/2)| = \sin(\delta_3/2)$. The sign of $|\sin[(\delta_3 - \delta_2)/2]|$ can be determined by comparing the values of E_{12} and E_{13}. For example, if $E_{13} > E_{12}$, it can be obtained that $\sin(\delta_3/2) > \sin(\delta_2/2)$ and $\delta_3/2 > \delta_2/2$, and then $|\sin[(\delta_3 - \delta_2)/2]| = \sin[(\delta_3 - \delta_2)/2]$. Thus, Eq. (6) can be rewritten as

$$E_{12} = C \times \sin(\delta_2/2), E_{13} = C \times \sin(\delta_3/2), E_{23} = C \times \sin\left[(\delta_3 - \delta_2)/2\right] \tag{7}$$

Since E_{12}, E_{13} and E_{23} can be determined, we get the following relationship which relates E_{12} etc., with the parameter C,

$$E_{13} = E_{12} \times \sqrt{1 - (E_{23}/C)^2} + E_{23} \times \sqrt{1 - (E_{12}/C)^2} \tag{8}$$

After solving for C, δ_2 and δ_3 can be directly calculated from E_{12} and E_{13}.

Once the phase shifts $\delta_k (k = 2, 3)$ are known, the wrapped phase φ can be solved with following expression:

$$\varphi = \arctan\frac{I_3 - I_2 + (I_1 - I_3)\cos\delta_2 + (I_2 - I_1)\cos\delta_3}{(I_1 - I_3)\sin\delta_2 + (I_2 - I_1)\sin\delta_3} \tag{9}$$

Here, the pixel coordinates have been omitted for simplicity.

3. Numerical simulations

A series of numerical simulations of three-frame GPSI have been carried out to verify the effectiveness of the method proposed above.

First, we tested the method with two kinds of fringe patterns: the closed ring fringe pattern and the open straight fringe pattern. For the ring fringe pattern, the background intensity and the modulation amplitude are set as $a_{mn} = 120 \times \exp\{-[m^2 + n^2]/500^2\}$ and $b_{mn} = 100 \times \exp\{-[m^2 + n^2]/500^2\}$ respectively. The measured phase is set as $\varphi_{mn} = -\pi \times N_f \times [m^2 + n^2]/200^2$, in which $N_f = 2$ is the fringe number in the fringe pattern, $m, n = -500, -499, \ldots, 500$. The phase shift values of the 1th, 2th and 3th fringe patterns are preset as $\delta_1 = 0$ rad, $\delta_2 = 0.1$ rad and $\delta_3 = 0.2$ rad, respectively. Moreover, Gaussian noise with a signal-to-noise ratio (SNR) of 30 dB is added to the fringe pattern. With above parameter setting, three simulated patterns with the size of 1000×1000 pixels can be generated with Eq. (1), as shown in Fig. 1(a–c). By means of numerical calculation, δ_2 and δ_3 are determined as 0.0960 rad and 0.1907 rad with the associated errors of 0.0040 rad and −0.0093 rad, respectively. Then, the wrapped and unwrapped phase maps can be obtained easily after the determination of the phase shift, which are illustrated in Fig. 1(d) and (e), respectively. Fig. 1(f) is the theoretical phase. From Fig. 1(e) and (f), no significant difference can be observed.