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A spatial algorithm to reduce phase wraps from two dimensional signals in fringe projection profilometry

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ABSTRACT

In this paper, we present a novel algorithm to reduce the number of phase wraps in two dimensional signals in fringe projection profilometry. The technique operates in the spatial domain, and achieves a significant computational saving with regard to existing methods based on frequency shifting. The method works by estimating the modes of the first differences distribution in each axial direction. These are used to generate a tilted plane, which is subtracted from the entire phase map. Finally, the result is re-wrapped to obtain a phase map with fewer wraps. The method may be able to completely eliminate the phase wraps in many cases, or can achieve a significant phase wrap reduction that helps the subsequent unwrapping of the signal. The algorithm has been exhaustively tested across a large number of real and simulated signals, showing similar results compared to approaches operating in the frequency domain, but at significantly lower running times.

1. Introduction

Phase measuring fringe projection profilometry methods attempt to obtain a surface measurement by projecting a sinusoidal fringe pattern onto an object. The height of the object at each point is related to the phase of the deformed fringe pattern. This is usually extracted by using phase retrieval methods [1,2], such as Fourier [3], wavelet [4], FIR Hilbert transformers [5], Shearlet transforms [6] or phase shifting methods [7].

In all these cases, the retrieved information consists of the principal values, which are bounded in the range $(-\pi, \pi]$. This may cause discontinuities within the phase signal (wraps), which should not be present in the unbounded height measurements. The process of resolving these phase discontinuities is called phase unwrapping, and the process also occurs in various other fields of research, such as in geoscience (SAR radar) and in medicine (MRI scanning). Phase unwrapping has been a matter of extensive research for the last two decades. As a result, many different techniques have been developed to tackle phase unwrapping problems. These include global error minimization algorithms [8,9], branch-cut methods [10,11], quality guided techniques [12–15] and region-growing approaches [16,17].

Attempts have also been made at reducing the number of fringes in the phase map, to reduce the complexity of the problem. In Fourier profilometry, one way to achieve this is by frequency shifting in the Fourier domain [3,18]. This approach has been recently extended so that it can be used with other phase retrieval methods [19]. The wrap reduction has a positive effect on the subsequent unwrapping step. However, the method suffers in terms of execution speed from the significant computation involved in the necessary trigonometric operations and the 2D discrete Fourier transform (DFT) that are involved.

In this paper, we present an alternative algorithm that operates in the spatial domain and hence does not require the use of the DFT. The technique resembles the behavior of the frequency shift method [19]. First, the modes of the first differences distribution in each direction are estimated. These are then used to generate a tilted plane, which is subtracted from the entire phase map. Finally, the result is re-wrapped to obtain a phase map that contains fewer wraps. The wrap reduction that is achieved by the proposed approach is similar to that which is obtained by using the frequency shift method, but the new method runs in linear time, so is significantly faster than the method presented in [19]. Potential benefits to unwrapping results are illustrated on both simulated and real fringe patterns.

2. Wrap reduction in the Fourier domain

Given a wrapped phase signal $\phi_w(x,y)$, the method presented in [19] applies Algorithm 1 to reduce the number of wraps in the

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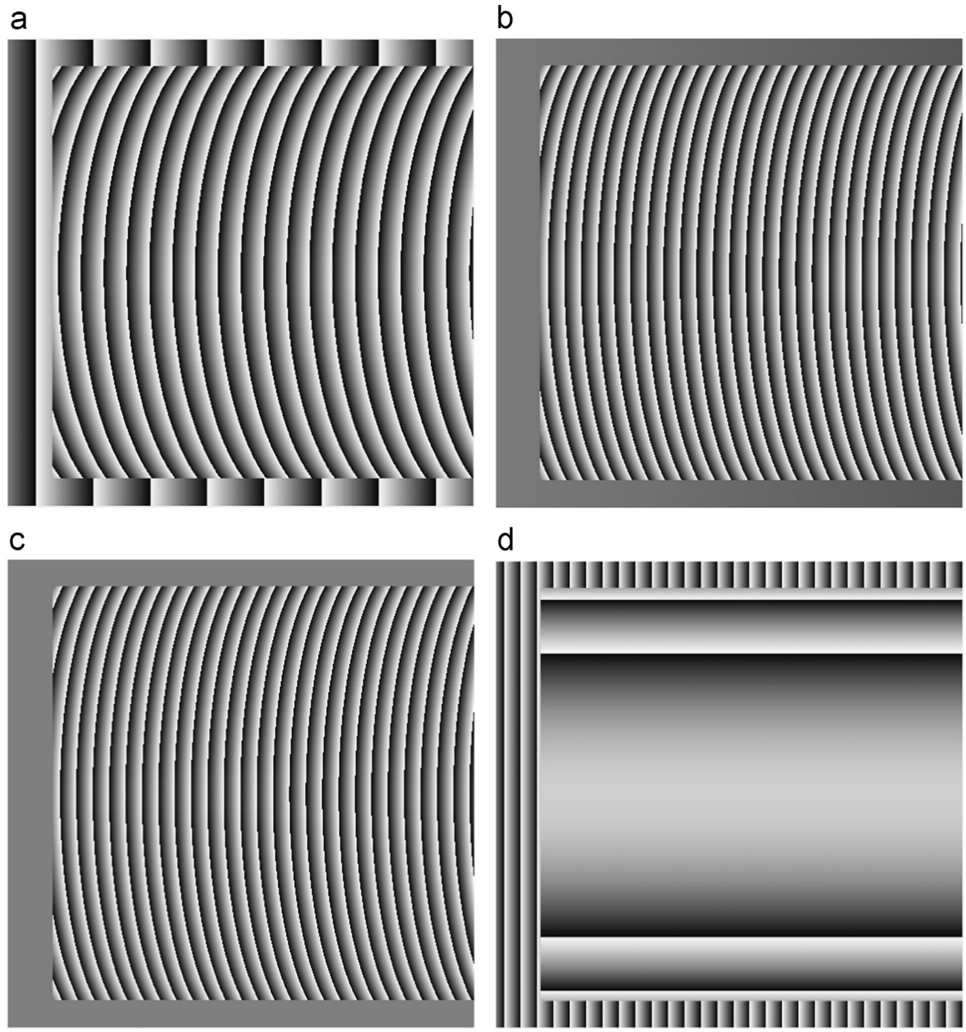

Fig. 1. (a) A simulated wrapped phase signal, and results obtained by (b) frequency shifting in Fourier space; (c) spatial method using bivariate distribution; and (d) spatial method considering each axial direction separately.

phase map. First, the complex signal $\psi(x,y)$ is built, and the 2D Fourier transform operator $\mathcal{F}$ is applied to the result. Then, the indexes for the maximum value in the frequency spectrum, u_0 and v_0, are determined. These are used to shift the Fourier transform of the complex signal towards the origin. Subsequently, the inverse Fourier transform of the shifted signal $\Psi'(u,v)$ is computed, to yield $\psi'(x,y)$. Finally, the new phase function $\phi'_w(x,y)$ is extracted by using the four quadrant arctan function on the ratio between the real and imaginary parts of signal $\psi'(x,y)$.

Algorithm 1. Pseudo-code for the wrap reduction algorithm in the Fourier Domain [19].

(1) $\psi(x,y) = e^{i\phi_w(x,y)}$;
(2) $\Psi(u,v) = \mathcal{F}\{\psi(x,y)\}$;
(3) $u_0, v_0 = \arg\max_{u,v} abs(\Psi(u,v))$;
(4) $\Psi'(u,v) = \Psi(u+u_0, v+v_0)$;
(5) $\psi'(x,y) = \mathcal{F}^{-1}\{\Psi'(u,v)\}$;
(6) $\phi'_w(x,y) = \arctan\left(\frac{\mathcal{I}(\psi'(x,y))}{\mathcal{R}(\psi'(x,y))}\right)$.

When using Fourier Analysis, the function $\Psi(u,v)$ and steps (3)–(6) in Algorithm 1 are an inherent part of the phase retrieval operation. Although the slight loss of accuracy has made some authors ignore the frequency shifting stage in later work, it was actually a step in the original method [18]. In addition, if the shift is combined with carefully designed optics, the technique has the potential to make unwrapping unnecessary [18].

When using other existing phase extraction methods, Algorithm 1 needs to be run entirely. Despite the apparent complexity of the method, it translates into the spatial domain as a removal of a tilted plane, followed by a re-wrap operation to the original bound interval. The slopes of this plane in the x and y directions are $2\pi u_0/N_X$ and $2\pi v_0/N_Y$, respectively, where N_X and N_Y refer to the size of the original wrapped phase signal.

This can be proven by considering the shift property of the Fourier transform: $f(x,y)e^{i2\pi(u_0x/N_X + v_0y/N_Y)} \Longleftrightarrow F(u-u_0, v-v_0)$. The function $\psi'(x,y)$ then becomes

$$\psi'(x,y) = \psi(x,y)e^{-i(2\pi u_0 x/N_X + 2\pi v_0 y/N_Y)} = e^{i(\phi_w(x,y) - 2\pi u_0 x/N_X - 2\pi v_0 y/N_Y)} \quad (1)$$

and the resulting phase map is given by

$$\begin{aligned}\phi'_w(x,y) &= \arctan\left(\frac{\mathcal{I}(\psi'(x,y))}{\mathcal{R}(\psi'(x,y))}\right)\\ &= \arctan\left(\frac{\sin(\phi_w(x,y) - 2\pi u_0 x/N_X - 2\pi v_0 y/N_Y)}{\cos(\phi_w(x,y) - 2\pi u_0 x/N_X - 2\pi v_0 y/N_Y)}\right)\\ &= \mathcal{W}(\phi_w(x,y) - 2\pi u_0 x/N_X - 2\pi v_0 y/N_Y) \quad (2)\end{aligned}$$

where $\mathcal{W}$ stands for the wrapping operator, as a result of the application of the arctan function; and $\mathcal{R}$ and $\mathcal{I}$ are used to refer