Optics and Lasers in Engineering 82 (2016) 135–140

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Generation of vector beams using a double-wedge depolarizer: Non-quantum entanglement

C.T. Samlan, Nirmal K. Viswanathan

School of Physics, University of Hyderabad, Hyderabad 500046, India

ARTICLE INFO

Article history:
Received 10 November 2015
Received in revised form
20 January 2016
Accepted 19 February 2016

Keywords:
Vector beam
Double wedge depolarizer
Non-quantum entanglement
Polarization interference

ABSTRACT

Propagation of horizontally polarized Gaussian beam through a double-wedge depolarizer generates vector beams with spatially varying state of polarization. Jones calculus is used to show that such beams are maximally nonseparable on the basis of even (Gaussian)–odd (Hermite–Gaussian) mode parity and horizontal–vertical polarization state. The maximum nonseparability in the two degrees of freedom of the vector beam at the double wedge depolarizer output is verified experimentally using a modified Sagnac interferometer and linear analyser projected interferograms to measure the concurrence 0.94 ± 0.002 and violation of Clauser–Horne–Shimony–Holt form of Bell-like inequality 2.704 ± 0.024. The investigation is carried out in the context of the use of vector beams for metrological applications.

1. Introduction

The exploitation of mathematical isomorphism between paraxial wave theory of light and quantum mechanics is on the rise in the last couple of decades, starting from the Pancharatnam–Berry geometric phase [1,2] and weak measurements [3,4] to spin Hall Effect of light – SHEL [5] and hybrid entanglement (nonseparable degrees of freedom) in classical light beams. Of interest to us here, entanglement is classified into that between spatially separated systems (non-local) [6] and between different degrees of freedom (DoF) of a single system (local) [7]. Spreeuw in 1998 showed that entanglement is possible between position and polarization DoF in an optical beam and named it *'classical entanglement'* to differentiate it from its quantum counterpart [7]. More recently, Qian and Eberly identified the degree of polarization (DoP) of an optical field as a measure of degree of entanglement, wherein the field amplitude and polarization vector are the two entangled DoF [8]. This was later extended to demonstrate violation of Clauser–Horne–Shimony–Holt (CHSH) form of Bell's inequality in classical optical fields [9]. Entanglement between polarization and spatial mode DoF were also verified and quantified by different quantum-inspired measurements like entropy, concurrence and violation of Bell-like inequalities [10–14].

Applications of the classical entangled optical beams includes fundamental areas like the resolution of a long-standing issue concerning Mueller matrices [15], an alternative interpretation of DoP [8] and Bell's measure as a new index of optical coherence [16]. The proposals for using classical entangled beams in polarization metrology [17] and experimental demonstration of high-speed kinematic sensing using it [18] are some recently reported examples. Quantum computation algorithms [19], teleportation protocols [20,21] and quantum cryptography [22] were also proposed and experimentally realized using classically entangled optical beams.

Polarization interferometers [8], Pancharathnam–Berry phase optical elements (Q-plates) [23] and polarization sensitive spatial light modulators [16] are some of the techniques used for the generation of vector beams with nonseparable polarization and spatial mode DoF. These methods have their own advantages but lack of portability and expensiveness are some issues in their widespread commercial adaptability. Here we present the generation of vector beam with maximally nonseparable (MNS) polarization and spatial mode parity DoF using an off-the-shelf optical component, a double wedge depolarizer (DWD) suitable for high power beams and single photon applications. The DWDs though have been used predominantly to generate depolarized light [24], as the name suggests, we present here that under appropriate conditions a linearly polarized light beam passing through it can generate an optical beam with spatially periodic state of polarization (SoP), a vector beam. By varying the SoP of the input beam we can generate a vector beam with different variations in the output beam SoP and also a scalar beam, with spatially uniform polarization. The vector beam with nonseparable DoF generated using DWD is envisioned first via Jones matrix calculations and verified experimentally using concurrence measurement and is also shown to violate CHSH form of Bell-like

E-mail address: nirmalsp@uohyd.ac.in (N.K. Viswanathan).

http://dx.doi.org/10.1016/j.optlaseng.2016.02.016
0143-8166/

inequality, making the vector beam generation method simple and desirable for several metrological applications.

2. Theory

The geometry of the DWD is shown in Fig. 1. The beam propagation direction, z-axis is perpendicular to the face of the device. The thickness d_1 and d_2 of the wedges varies along the x-axis with $d_1+d_2=L$, the device thickness. The optic axes of the wedges are oriented at $+45°$ and $-45°$ with respect to y-axis in the x–y plane. The Jones matrix of the DWD, J_{DWD} can be obtained from J_2J_1, where J_1 and J_2 are the Jones matrices of the birefringent wedges 1 and 2 given by $J_1=\begin{pmatrix}1+e^{i\delta_1} & 1-e^{i\delta_1}\\ 1-e^{i\delta_1} & 1+e^{i\delta_1}\end{pmatrix}$ and $J_2=\begin{pmatrix}1+e^{i\delta_2} & e^{i\delta_2}-1\\ e^{i\delta_2}-1 & 1+e^{i\delta_2}\end{pmatrix}$[25]. Then,

$$J_{DWD}=\begin{pmatrix}e^{i\delta_2}+e^{i\delta_1} & e^{i\delta_2}-e^{i\delta_1}\\ e^{i\delta_2}-e^{i\delta_1} & e^{i\delta_2}+e^{i\delta_1}\end{pmatrix} \tag{1}$$

where δ_1 and δ_2 are the phase difference between x and y components of the field after propagating through the two wedges.

The Jones vector of a linearly polarized laser beam propagating in z-direction is,

$$\vec{E}(r)=E(r)\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \tag{2}$$

where $E(r)$ is the Gaussian amplitude $\left(e^{-r^2/\omega_0{}^2}\right)$ of the field and θ gives the direction of linear polarization of the beam. The beam after propagating through the DWD is given by,

$$\vec{E}'(r)=E(r)\begin{pmatrix}\cos(\theta)\cos\left(2\alpha\left(x-\frac{L}{2}\right)\right)+i\,\sin(\theta)\sin\left(2\alpha\left(x-\frac{L}{2}\right)\right)\\ i\,\cos(\theta)\sin\left(2\alpha\left(x-\frac{L}{2}\right)\right)+\,\sin(\theta)\cos\left(2\alpha\left(x-\frac{L}{2}\right)\right)\end{pmatrix}e^{i\alpha L} \tag{3}$$

Here, $\alpha=\frac{1}{2}k\Delta n\,\tan\chi$ where k is the propagation vector of the beam in free space, Δn is birefringence of the crystal, χ is the wedge angle (Fig. 1) and the range of x coordinate is constrained within the beam waist of the input beam.

Eq. (3) clearly shows that the state of polarization of the output beam varies sinusoidally with a period, $\Lambda=\frac{\pi}{\alpha}$ in the x-direction for all input polarization states except for diagonal $(\theta=\frac{\pi}{4})$ and anti-diagonal $(\theta=-\frac{\pi}{4})$. For horizontally polarized $(\theta=0)$ input beam,

$$\vec{E}'(r)=E(r)\begin{pmatrix}\cos\left(2\alpha(x-\frac{L}{2})\right)\\ i\,\sin\left(2\alpha(x-\frac{L}{2})\right)\end{pmatrix}e^{i\alpha L} \tag{4}$$

This equation can be rewritten in the form,

$$\vec{E}'(r)=E_e(r)\hat{x}+iE_o(r)\hat{y} \tag{5}$$

where E_e and E_o are the spatially modulated Gaussian modes due to the terms $\cos\left(2\alpha(x-\frac{L}{2})\right)$ and $\sin\left(2\alpha(x-\frac{L}{2})\right)$ which are even and odd sinusoidal functions, respectively. Unit vectors $\hat{x}$ and $\hat{y}$ are the orthonormal polarization vectors corresponding to horizontal, $\begin{pmatrix}1\\0\end{pmatrix}$ and vertical, $\begin{pmatrix}0\\1\end{pmatrix}$ polarization states, respectively. One can approximate the sinusoidal functions to, $\cos\left(2\alpha(x-\frac{L}{2})\right)=1$ and $\sin\left(2\alpha(x-\frac{L}{2})\right)=2\alpha(x-\frac{L}{2})$ for smaller values of x, i.e., with smaller size of beam waist. So the even and odd spatial modes can be expressed in terms of 0th order (fundamental) and 1st order Hermite–Gaussian modes if the input beam waist is sufficiently small as stated in Eq. (6). We calculated a threshold beam waist for this approximation from numerically simulated results which is found to be, $\omega_0<\frac{\pi}{16\alpha}$.

$$E_e(r)=1e^{-r^2/\omega_0{}^2}=H_0\left[\alpha(x-\tfrac{L}{2})\right]e^{-r^2/\omega_0{}^2}$$
$$E_o(r)=2\alpha(x-\tfrac{L}{2})e^{-r^2/\omega_0{}^2}=H_1\left[\alpha(x-\tfrac{L}{2})\right]e^{-r^2/\omega_0{}^2} \tag{6}$$

where H_0 and H_1 are the zero and first order Hermite polynomials.

The nonseparability between polarization and spatial mode parity DoF in the output beam as described by Eq. (5) is very significant. The polarization and spatial mode parity DoFs considered as vectors belonging to two independent Hilbert vector spaces. Then, Eq. (5) can be considered as a superposition of products of vectors from different vector space that cannot be rearranged into a single product that separates the vectors, the reason for their nonseparability. We normalize Eq. (5) with respect to intensity $I=\langle E_e(r)E_e(r)+E_o(r)E_o(r)\rangle$ and remove the imaginary term by introducing a $\pi/2$ phase using a vertically oriented quarter wave plate (QWP) to give,

$$\vec{e}'(r)=\vec{E}'(r)/\sqrt{I}=e_e(r)\hat{x}+e_o(r)\hat{y} \tag{7}$$

We use quantum mechanical notations for simplicity in further equations. The unit polarization vectors $\hat{x}$ and $\hat{y}$ will be written as $\hat{x}\to|u_1\rangle$ and $\hat{y}\to|u_2\rangle$ and the unit spatial mode parity as $e_e(r)\to|f_1\rangle$ and $e_o(r)\to|f_2\rangle$ [9]. Here, $\langle u_1|u_2\rangle=\langle f_1|f_2\rangle=0$ and the projectors in the two spaces $|u_1\rangle\langle u_1|+|u_2\rangle\langle u_2|=|f_1\rangle\langle f_1|+|f_2\rangle\langle f_2|=1$. Then the output beam given by Eq. (7) takes the form,

$$\vec{E}'(r)/\sqrt{I}=|\mathrm{e}\rangle=\frac{1}{\sqrt{2}}\left(|u_1\rangle|f_1\rangle+|u_2\rangle|f_2\rangle\right) \tag{8}$$

In this notation the above equation is akin to a Bell state which then leads to a quick check of the correlation between the nonseparable DoF. Arbitrary rotation of the unit polarization vectors $|u_m\rangle$ and spatial mode parity functions $|f_n\rangle(m,n=1,2)$ through angles a and b respectively gives the correlation coefficients $C(a,b)$. Rotation through angles a and b is carried out using the rotation matrices:

$$\begin{pmatrix}|u_1^a\rangle\\ |u_2^a\rangle\end{pmatrix}=\begin{pmatrix}\cos(a) & -\sin(a)\\ \sin(a) & \cos(a)\end{pmatrix}\begin{pmatrix}|u_1\rangle\\ |u_2\rangle\end{pmatrix} \tag{9}$$

$$\begin{pmatrix}|f_1^b\rangle\\ |f_2^b\rangle\end{pmatrix}=\begin{pmatrix}\cos(b) & -\sin(b)\\ \sin(b) & \cos(b)\end{pmatrix}\begin{pmatrix}|f_1\rangle\\ |f_2\rangle\end{pmatrix} \tag{10}$$

Now the correlation between the polarization and mode parity function DoF is given by the average [9],

$$C(a,b)=\langle\mathbf{e}|Z^u(a)\otimes Z^f(b)|\mathbf{e}\rangle \tag{11}$$

where $Z^u(a)=|u_1^a\rangle\langle u_1^a|-|u_2^a\rangle\langle u_2^a|$ and $Z^f(b)=|f_1^b\rangle\langle f_1^b|-|f_2^b\rangle\langle f_2^b|$ are the difference projections corresponding to polarization and spatial mode parity DoF. Thus $C(a,b)$ is given by a combination of four joint projections,

$$C(a,b)=P_{11}(a,b)+P_{22}(a,b)-P_{12}(a,b)-P_{21}(a,b) \tag{12}$$

where

$$P_{11}(a,b)=\langle\mathbf{e}||u_1^a\rangle|f_1^b\rangle\langle f_1^b|\langle u_1^a||\mathbf{e}\rangle=\left|\langle f_1^b|\langle u_1^a||\mathbf{e}\rangle\right|^2$$

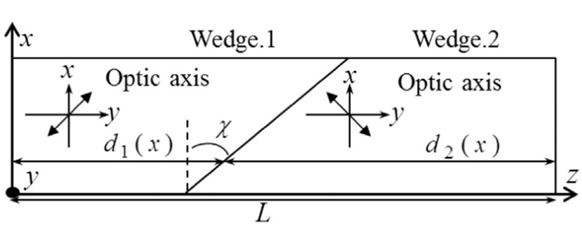

Fig. 1. Schematic diagram of the DWD. The two optic axes are oriented in 45° and −45° to y-axis.