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Phase extraction from arbitrary phase-shifted fringe patterns with noise suppression

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A B S T R A C T

Extracting phase distribution from arbitrary phase-shifted fringe patterns, if possible, is very useful in phase-shifting interferometry. The advanced iterative algorithm (AIA) is introduced and the windowed Fourier ridges and least squares fitting (WFRLSF) is proposed. Both algorithms are sensitive to noise, which limits their applications to almost perfect fringe patterns. The windowed Fourier filtering (WFF) algorithm is proposed for both pre-filtering and post-filtering to suppress the noise. Simulation results show that with the effective noise suppression, the phase error is reduced to less than 0.1 rad. Experimental examples are also given for verification. The almost identical results produced by the AIA and the WFRLSF suggest that both algorithms can be used for phase extraction with cross-validation.

1. Introduction

Phase-shifting interferometry is a powerful technique for full-field, accurate and non-contact measurements [1–6]. To extract phase distribution from fringe patterns with known phase-shifts, the accuracy of the phase-shifts is essential but often difficult to be guaranteed [1–4]. Consequently error-compensating phase-shifting algorithms were proposed, assuming that the phase-shifts are around their nominal values [5,6]. This solution is straightforward and has been well accepted. An alternative solution is to extract the phase distribution without knowing the phase-shift values. One example is the advanced iterative algorithm (AIA) proposed by Wang and Han [7]. The AIA evolved from Okada et al.'s work [8] which was extended from Greivenkamp's work [9]. The AIA has attracted some research interests recently due to its high effectiveness [10–12]. Another method called WFRLSF is proposed in this paper. It first estimates phase-shifts by a windowed Fourier ridges (WFR) algorithm [13] and then estimates phase distribution by a least squares fitting (LSF) [7–9].

For the AIA, noise affects the convergence of iterations and consequently the accuracy of the extracted phase. For the WFRLSF, though the WFR algorithm is able to accurately estimate the phase-shifts from noisy fringe patterns, the subsequently extracted phase is still noisy. It will be seen that in both algorithms, the phase error increases with noise level. To improve immunity to noise, a windowed Fourier filtering algorithm (WFF) [14–16] is proposed for both pre-filtering before phase extraction and post-filtering after phase extraction. Simulation shows that with the noise suppression, both the AIA and the WFRLSF produce phase errors less than 0.1 rad. It will be observed, interestingly and surprisingly, that the AIA and the WFRLSF give almost identical results, though their principles are quite different. Thus the WFRLSF, having no convergence problem, can serve as an empirical way to show the convergence of the AIA which has not been proven yet.

The contributions of this paper include

(i) The WFRLSF for phase extraction from arbitrary phase-shifted fringe patterns is proposed;
(ii) The WFF which will be shown to be very effective as pre-filtering and post-filtering for both the AIA and the WFRLSF is proposed;
(iii) The WFRLSF can be used to show the convergence of the AIA due to their very similar performances.

The rest of the paper is organized as follows. The AIA is introduced in Section. 2. The WFF and the WFR are introduced in Section 3. The WFRLSF is proposed in Section 4. Using the WFF to improve the performances of the AIA and the WFRLSF is proposed in Section 5. Simulated and experimental examples are given for verification. Spatially non-uniform phase-shifts and high order harmonics as error sources in phase extraction are discussed. The paper is concluded in Section 6.

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2. Advanced iterative algorithm (AIA) [7]

The AIA introduced in this section is directly adapted from Ref. [7] for completeness of this paper. The notations are similar to those in [7] for consistency. A part of the AIA, Eqs. (4–5), will also be used for the WFRLSF proposed in Section 4.

Phase-shifted fringe patterns can be written as follows:

$$f_{ij} = A_{ij} + B_{ij}\cos(\varphi_j + \delta_i) + n_{ij}, \tag{1}$$

where the subscript $i=1,2,\ldots,M$ is used as a frame index and M is the total frame number; the subscript $j=1,2,\ldots,N$ is used as a pixel index and N is the total pixel number; f_{ij}, A_{ij}, B_{ij} and n_{ij} are fringe intensity, background intensity, fringe amplitude, and noise of frame i and pixel j, respectively; φ_j is the phase value of pixel j; and δ_i is the phase-shift value of frame i.

Eq. (1) can be rewritten as

$$f_{ij} = a_j + b_j\cos\delta_i + c_j\sin\delta_i + n_{ij}, \tag{2}$$

where $a_j = A_{ij}$, $b_j = B_{ij}\cos\varphi_j$ and $c_j = -B_{ij}\sin\varphi_j$ are assumed to be constant in different frames. If the phase-shifts δ_i are known, for each pixel j, a_j, b_j and c_j can be solved from Eq. (2) by the LSF, i.e., by minimizing the residual error

$$E_j = \sum_{i=1}^{M} (a_j + b_j\cos\delta_i + c_j\sin\delta_i - f_{ij})^2, \tag{3}$$

which leads to the following simplified calculation:

$$\begin{bmatrix} a_j \\ b_j \\ c_j \end{bmatrix} = \begin{bmatrix} M & \sum_{i=1}^{M}\cos\delta_i & \sum_{i=1}^{M}\sin\delta_i \\ \sum_{i=1}^{M}\cos\delta_i & \sum_{i=1}^{M}\cos^2\delta_i & \sum_{i=1}^{M}\cos\delta_i\sin\delta_i \\ \sum_{i=1}^{M}\sin\delta_i & \sum_{i=1}^{M}\cos\delta_i\sin\delta_i & \sum_{i=1}^{M}\sin^2\delta_i \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{M} f_{ij} \\ \sum_{i=1}^{M} f_{ij}\cos\delta_i \\ \sum_{i=1}^{M} f_{ij}\sin\delta_i \end{bmatrix}, \tag{4}$$

where the superscript -1 denotes the matrix inverse. The phase can then be calculated as

$$\varphi_j = \tan^{-1}(-c_j/b_j). \tag{5}$$

Similarly Eq. (1) can be rewritten as

$$f_{ij} = a'_i + b'_i\cos\varphi_j + c'_i\sin\varphi_j + n_{ij}, \tag{6}$$

where $a'_i = A_{ij}$, $b'_i = B_{ij}\cos\delta_i$ and $c'_i = -B_{ij}\sin\delta_i$ are assumed to be constant for all the pixels. If the phase values of φ_j are known, for each frame i, a'_i, b'_i and c'_i can be solved from Eq. (6) by the LSF, i.e., by minimizing the residual error

$$E'_i = \sum_{j=1}^{N} (a'_i + b'_i\cos\varphi_j + c'_i\sin\varphi_j - f_{ij})^2, \tag{7}$$

which leads to the following simplified calculation:

$$\begin{bmatrix} a'_i \\ b'_i \\ c'_i \end{bmatrix} = \begin{bmatrix} N & \sum_{j=1}^{N}\cos\varphi_j & \sum_{j=1}^{N}\sin\varphi_j \\ \sum_{j=1}^{N}\cos\varphi_j & \sum_{j=1}^{N}\cos^2\varphi_j & \sum_{j=1}^{N}\cos\varphi_j\sin\varphi_j \\ \sum_{j=1}^{N}\sin\varphi_j & \sum_{j=1}^{N}\cos\varphi_j\sin\varphi_j & \sum_{j=1}^{N}\sin^2\varphi_j \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^{N} f_{ij} \\ \sum_{j=1}^{N} f_{ij}\cos\varphi_j \\ \sum_{j=1}^{N} f_{ij}\sin\varphi_j \end{bmatrix}, \tag{8}$$

The phase-shifts can then be calculated as

$$\delta_i = \tan^{-1}(-c'_i/b'_i). \tag{9}$$

Both phase values and phase-shifts are estimated by alternately using Eqs. (4 and 5) and Eqs. (8 and 9), which is iterated until a certain condition is satisfied [7]. This forms the AIA algorithm. Its convergence is not theoretically proven. However, the method works well especially when the noise level is very low, which is evident from Ref. [7] and subsequent publications [10–12].

3. Windowed Fourier transform [14–16]

In this section, the windowed Fourier ridges (WFR) and the windowed Fourier filtering (WFF), two algorithms based on windowed Fourier transform, are briefly introduced. They are directly adapted from Refs. [14–16] for the completeness of this paper. The notations are similar to those in [14–16] for consistency. The WFR will be used in developing the WFRLSF, while the WFF will be adopted to suppress noise in both the AIA and the WFRLSF.

A windowed Fourier transform pair can be expressed as a forward and an inverse transforms as follows:

$$Sf(u,v;\xi,\eta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, g^*_{u,v;\xi,\eta}(x,y)\,dx\,dy, \tag{10}$$

$$f(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Sf(u,v;\xi,\eta)\, g_{u,v;\xi,\eta}(x,y)\,d\xi\,d\eta\,du\,dv, \tag{11}$$

where $f(x,y)$ is an input fringe pattern; $Sf(u,v;\xi,\eta)$ is the windowed Fourier spectrum of $f(x,y)$; (x,y) and (u,v) are spatial coordinates; (ξ,η) is a frequency coordinate; the symbol * denotes a complex conjugate operator; the windowed Fourier element $g_{u,v;\xi,\eta}(x,y)$ is a windowed harmonic that is spatially centered at (u,v) and tuned to a frequency of (ξ,η).

The WFR algorithm searches for a windowed Fourier element which is the most similar to each portion of a fringe pattern covered by the window. Consequently, frequencies of the fringe pattern at (u,v) along x and y directions, $\omega_x(u,v)$ and $\omega_y(u,v)$, can be determined as

$$[\omega_x(u,v), \omega_y(u,v)] = \arg\max_{\xi,\eta} |Sf(u,v;\xi,\eta)|, \tag{12}$$

where $\arg\max_{\xi,\eta}$ means that the arguments ξ and η which maximize $|Sf(u,v;\xi,\eta)|$ are taken as $\omega_x(u,v)$ and $\omega_y(u,v)$, respectively. The local frequency $[\omega_x(u,v), \omega_y(u,v)]$ is also called the ridge location. Subsequently phase distribution can be extracted from the ridge information:

$$\varphi(u,v) = \text{angle}\{Sf[u,v;\omega_x(u,v),\omega_y(u,v)]\} + \omega_x(u,v)u + \omega_y(u,v)v. \tag{13}$$

When the WFR is used to process a single fringe pattern, it should be highlighted that

(i) The WFR gives results with sign ambiguity, i.e., if $\omega_x(u,v)$, $\omega_y(u,v)$ and $\varphi(u,v)$ is a solution, so is $-\omega_x(u,v)$, $-\omega_y(u,v)$ and $-\varphi(u,v)$;
(ii) The WFR does not give accurate estimation when both $|\omega_x(u,v)|$ and $|\omega_y(u,v)|$ are low.

The WFF algorithm is another algorithm based on the windowed Fourier transform, which assumes that the windowed Fourier spectrum of noise in a fringe pattern permeates the entire windowed Fourier domain with small spectrum coefficients. The WFF algorithm hence filters a fringe pattern by thresholding its windowed Fourier spectrum, which can be written as

$$\overline{f}(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\eta_l}^{\eta_h}\int_{\xi_l}^{\xi_h} \overline{Sf}(u,v;\xi,\eta)\, g_{u,v;\xi,\eta}(x,y)\,d\xi\,d\eta\,du\,dv, \tag{14}$$