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Optical transfer function of an optical system with spiral zone masks in presence of primary aberrations

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ABSTRACT

The optical transfer function (OTF), of an optical system with spiral phase zone plate (SPZP) masks with integer and fractional topological charge has been analyzed and compared with the OTF of a system with a vortex phase mask. The OTF of an optical system with vortex mask becomes negative in a certain frequency range, while that of a system with spiral phase zone plate mask shows different characteristics and attains negative values in multiple frequency bands. The effect of primary aberrations on the characteristics of the OTF is analyzed and it has been shown that the effects of balanced astigmatism and balanced coma on OTF are asymmetric while those of balanced spherical aberration are observed to be symmetric.

1. Introduction

The optical transfer function (OTF) is normalized autocorrelation function of the amplitude transfer function (ATF), where the ATF is Fourier transform of the space invariant amplitude impulse response of the pupil function of an optical system [1]. The OTF and the point spread function (PSF) are Fourier transform pairs of each other. The OTF can also be considered as a straight application of the FT theory in which a two-dimensional deviation of the intensity over the object plane is analyzed into a two-dimensional spectrum of special frequencies and the image of incoherent extended objects can be determined by integrating the product of spectrum of the object and the OTF [2]. Recently the spiral vortex lens has also drawn interests in imaging, lithography, astronomy etc [3,4]. Out of these applications, one is photolithography where partially coherent beam is used to produce sharp structures in comparison to non vortex beams. Significant amount of literature is available on the OTF of conventional optical systems with and without aberrations [1,2,5–11]. The application of vortex mask in photolithography divests from the wavelength dependence in patterning by printing an array of holes in a negative resist and these contact holes are up to the fundamental limit on the pitch of 2D patterning. However, in most of the practical situations there is chance of residual aberrations. Although, the imaging in spatially incoherent light is unaffected by the vortex lens, all light being partially coherent, it opens the prospect to encode the vortices for low coherence light for many applications, viz. white light optical tweezers or incoherent encryption [3]. As the spiral zone lenses are known to produce a better (smaller and brighter) doughnut in comparison to the vortex phase mask, the application of spiral zone lens/plate phase mask, would be more convenient in patterning finer structures in photolithography. The spiral zone masks are also useful in image processing [12–16]. In lieu of the importance of the vortex beam in imaging, the OTF of an optical system with a vortex phase mask is studied in presence of primary aberrations by Singh et al. [11]. The OTF of such a system becomes negative in a certain frequency range even in the absence of aberrations thus showing the contrast reversal. Therefore, in this paper, we study the OTF of an optical system with spiral phase zone masks with fractional as well as with integer charge. We analyze the optical transfer function of an optical system with different spiral phase zone masks with and without aberrations. As for practical applications, the fabrication of a binary mask is easier and binary masks show better efficiency; we also consider a binary spiral phase zone plate (BSPZP) for the analysis of OTF.

2. Mathematical description

The optical transfer function of any optical system can be calculated either directly from the autocorrelation function of the pupil function by normalizing it with respect to the maximum value, or from the Fourier transform of the pupil function using the

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autocorrelation theorem. The spiral zone function can be written as [12,14]

$$g(r,\theta) = \exp \quad \{i(p\theta - \pi r^2/\lambda f)\} \times circ(r/R), \tag{1}$$

And the spiral zone mask carrying primary aberrations can be represented as

$$g(r,\theta) = \exp \quad \{i(p\theta - \pi r^2/\lambda f)\} \times \exp \quad \{ikW(r,\theta)\} \times circ(r/R) \tag{2}$$

where, p is the topological charge of the spiral phase in the function, f is the primary focal length of the zone function which is defined in a circular pupil function of radius R, k represents the propagation constant and $W(r,\theta)$represents the aberration function, which is given by [11]

$$W(r,\theta) = A_s r^4 + A_c r^3 \cos(\theta) + A_a r^2 \cos^2(\theta) + A_f r^2 + A_d r \cos(\theta) \tag{3}$$

where, A_s, A_c, A_a, $A_f$and A_d are coefficients for spherical aberration, coma, astigmatism, field curvature (or defocus) and tilt (or distortion), respectively.

The OTF calculation is performed by using the autocorrelation method. The autocorrelation of a function $g(x,y)$ can be expressed as

$$g(x,y) \odot g(x,y) = \iint g(\xi,\zeta) \times g^*(\xi - x, \zeta - y) d\xi d\zeta \tag{4}$$

We solve the autocorrelation integral numerically for the different spiral zone plate masked pupil functions and normalize them to get the OTF. Although, for brevity we present the OTF results from autocorrelation method, we obtain them using FT method also for cross-checking our calculations. The OTF characteristics in different conditions are discussed in the text to be followed.

3. OTF for different zone plate masks

3.1. Spiral phase zone plate (SPZP) masks with variable number of zones in the pupil

The spiral phase zone plate transmittance function is given in Eq. (1). The number of zones in the pupil can be varied by changing f parameter. It is known that the OTF of an optical system with spiral phase mask attains negative value is a certain frequency band [11]. The spiral phase zone plate consists of zones spiralling outwards from the centre. The optical transfer function of an optical system with SPZP significantly depends on the number of such spirals within the pupil of the system. We have analyzed the effect by considering different number of zones in the pupil function. It has been found that the OTF of such a system attains negative values in multiple frequency bands and the number of bands for which the OTF becomes negative depends on number of spirals (n) in the pupil. For n spirals in the pupil, the OTF becomes negative for $(2n-1)$ times. This is because of the number of phase jumps in a particular radial direction in the pupil. Therefore, the number of times, the OTF plot arrives at negative values provides an idea about the number of π phase jumps in the pupil function in the same direction. We have numerically calculated the OTF for an optical system with SPZP using autocorrelation method and the real value of the OTF is plotted along meridional radial direction as shown in Fig. 1. It can be seen that in all cases i.e. for different number of zones in the pupil, the OTF attains negative values in multiple frequency bands. The OTF for the SPZP with 1.5, 2, 3, 5 zones in the pupil attains negative values for 2, 3, 5, 9 frequency bands, respectively. Moreover, when the number of spirals is increased in the pupil, the OTF is seen squeezed towards the zero frequency regions. It happens because of the phase jumps present near the centre of the pupil for a SPZ lens with more number of spirals. Such a zone lens produces the contrast reversal also for very low frequencies, unlike a vortex mask pupil which only produces contrast reversal in one and relatively higher frequency band.

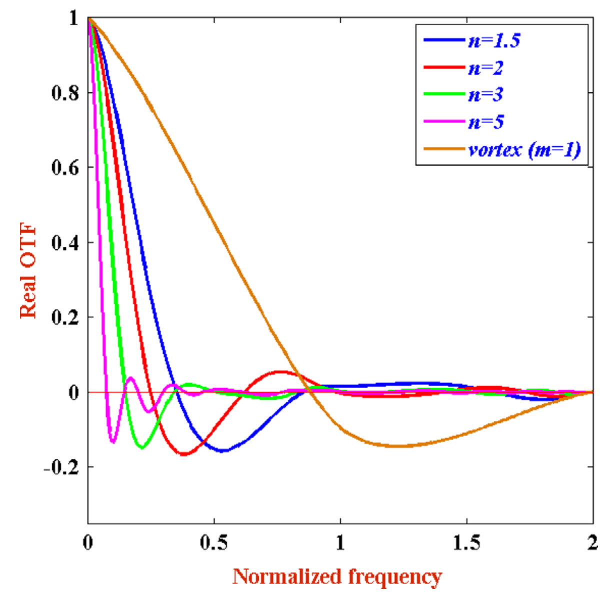

Fig. 1. Transverse OTF plot for spiral phase zone plate with variable number of zones in the pupil.

3.2. SPZP with different topological charges

The variation in the OTF, for an optical system whose pupil is consists of SPZP mask or vortex mask, with the topological charge, is shown in Fig. 2(a) and (b) for integer and fractional charges, respectively. We have considered the SPZP and vortex mask pupil functions with charges 1, 2, 3 and 5. The corresponding OTF plot, as shown in Fig. 2(a), reveals that the OTF for higher charge vortex masks becomes steeper around the zero frequency and consequently shows different frequency bands for which OTF attains a negative value. Moreover, the maximum negative value to which the OTF reaches is more for higher charges than for single charge, hence producing better contrast. Also the OTF becomes negative in more than one frequency bands for charge >2. In mid frequency region, higher charge vortex mask produces better contrast. It can be seen from the figure that at high frequencies, the OTF depicts a contrast reversal for odd charges, while for even higher charges no contrast reversal is seen at higher frequencies.

So far the OTF for the SPZP is concerned; it also squeezes around the zero frequency on increasing the topological charge. It can be seen from the Fig. 2(a) that the OTF attains more negative value on increasing the charge consequently, better contrast reversal is achievable. As the OTF dies fast for SPZP, the sharp features in the image would be seen at lower frequencies in comparison to the same charge vortex mask pupil functions.

Considering the fractional charge case, OTF of an optical system with fractional charge vortex shows some spectacular changes. When the charge becomes less than or equal to half, the OTF doesn't attain negative value, while for 1.5 charge OTF attains a larger negative value than for single charge vortex mask. For SPZP, the OTF shows regular characteristics as shown for multiple charges as can be seen from Fig. 2(b). Thus, the contrast reversal can be obtained in the image using a SPZP of even fractional charges less than 0.5 while the pupil with vortex phase shows no such contrast reversal.

3.3. Binary spiral phase zone mask (BSPZM)

Binary phase masks are often known to provide higher efficiency when they are used as filters or imaging systems. We have seen some drastic changes in the OTF behaviour when a BSPZP pupil is used in comparison to SPZP pupil. As can be seen from