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Unit root quantile autoregression testing with smooth structural changes☆

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ABSTRACT

By incorporating the flexible Fourier form into quantile autoregression model, this paper proposes three new unit root test statistics, which are robust to both non-Gaussian condition and structural changes. Since their limiting distributions are non-standard, a bootstrap procedure is developed to calculate their critical values. Monte Carlo simulation results show that, while Koenker and Xiao (2004) tests are quite conservative under various kinds of error distributions and structural changes, the newly proposed tests have good size performance except for a little size distortion occasionally. Moreover, our tests have much higher performance especially when the sample size is small.

1. Introduction

Testing unit root hypothesis is of particular interest to economists and finance practitioners since it has important policy implications. For example, the unit root property for the macroeconomic time series is closely related to the persistence of macroeconomic shocks. Nelson and Plosser (1982) investigated the unit root property of 14 U.S. macroeconomic time series and could not reject the null hypothesis of a unit root for any of them. Thus, they advocated that the source of business fluctuations was non-monetary. Sekioua (2006) studied the unit root property of the forward premium. If the forward premium is a unit root process, it makes the commonly employed market efficiency test inappropriate since it suffers from the spurious regression-type critique. The unit root tests also play fundamental role in checking the validity of financial theories and models, such as the purchasing power parity theory (Ma et al., 2017) and the Fed model (Koivu et al., 2005). However, most widely used unit root tests suffer from low power under the non-Gaussian condition (Koenker and Xiao, 2004; Li and Park, 2016) or when the trend function of the series has structural changes (Perron, 1989), which may induce misleading conclusions and policy implications. To deal with these issues, developing robust unit root test methods is imperative.

To improve the power performance under non-Gaussian condition, Koenker and Xiao (2004) developed three tests which are robust to various kinds of error distributions based on the quantile autoregression approach. Galvao (2009) extended their work by introducing stationary covariates and a linear time trend into the model. For power loss caused by structural changes in the deterministic components of the series, Perron (1989) modified the standard Dickey-Fuller (DF) test to allow for a single exogenous structural break at a prespecified change point. For subsequent developments along this line, see Perron (2006) for an excellent survey. Nevertheless, these tests usually depend crucially on the locations, numbers and the type of changes, which makes them inflexible and difficult to implement in practice. Recently, Enders and Lee (2012a) developed a new LM type unit root

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test by using the flexible Fourier form to account for structural changes. Rodrigues and Taylor (2012) and Enders and Lee (2012b) proposed the corresponding local GLS detrend type and DF type tests. Though power loss in unit root tests due to the departure from Gaussian condition and in the presence of structural changes has been separately studied, they had never been treated simultaneously. Given that most time series in economics and finance have notoriously heavy-tailed behavior and major events such as the global financial crisis could induce a level and/or slope change to the trend function of the time series, it is of great importance to develop new unit root tests which are robust to both non-Gaussian condition and various kinds of structural changes.

To the best of our knowledge, this paper is the first in the literature to study unit root tests which are robust to both non-Gaussian condition and structural changes in the trend function of a series. We achieve this by incorporating the flexible Fourier form into the quantile autoregression model and three new unit root test statistics, say, the quantile t-ratio test, the quantile Kolmogorov–Smirnov (QKS) test and the quantile Cramér-von Mises(QCM) tests are developed. Limiting distributions of these test statistics are derived and bootstrap procedures are developed to obtain their finite-sample critical values. Finally, Monte Carlo simulations are conducted to explore the finite sample performance of the newly proposed three tests.

2. Quantile autoregression with a Fourier function

Consider the following data generated process (DGP):

$$y_t = \alpha(t) + \gamma t + e_t, \quad e_t = \phi e_{t-1} + \mu_t, \tag{1}$$

where $\alpha(t)$ is a deterministic function of t representing potential level and/or slope changes in the trend function of the series, γt represents the deterministic trend, μ_t is assumed to be i.i.d with mean 0 and variance σ_μ^2 for simplicity. This model is first suggested by Schmidt and Phillips (1992) to avoid the ambiguous meaning of parameters existing in traditional Dickey–Fuller type unit root test model (also see Galvao, 2009. By employing this model, we can analyze the trending and unit root behavior of the series separately and in particular, we are mainly interested in testing the unit root null hypothesis (i.e., H_0: $\phi = 1$).

However, the functional form of $\alpha(t)$ is usually unknown in practice, which makes any test for H_0: $\phi = 1$ problematic and might induce misleading conclusion if it is misspecified. It is well documented that an autoregressive process with structural changes is often incorrectly regarded as a unit root process (Perron, 1989). Following Enders and Lee (2012a), we approximate it using the Fourier expansion with only a single frequency as it can often capture the essential characteristics of an unknown functional form:

$$\alpha(t) \cong \alpha_0 + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T). \tag{2}$$

Comparing to nonparametric or semi-parametric methods, this method is easy to implement and the convergence rates of parameters are faster. As Enders and Lee (2012a, 2012b) pointed out, this approximation method could help researchers avoid selecting specific endogenous break dates, the number of breaks, and the form of the breaks. The specification problem is transformed to incorporating the appropriate frequency components into the estimating equation. Using this approximation equation and rewriting Model (1), the testing procedure for the unit root null hypothesis could be based on the following model:

$$y_t = \phi\tilde{y}_{t-1} + \alpha_0 + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T) + \gamma t + \mu_t, \tag{3}$$

where $\tilde{y}_{t-1} = y_{t-1} - \alpha_0 - \alpha_k \sin(2\pi k(t-1)/T) - \beta_k \cos(2\pi k(t-1)/T) - \gamma(t-1)$. In particular, we are interested in testing unit root property at the τth quantile level of y_t:

$$Q_{y_t}(\tau|y_{t-1}) = \phi\tilde{y}_{t-1} + \alpha_0 + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T) + \gamma t + F_\mu^{-1}(\tau). \tag{4}$$

Though there are trigonometric terms, Model (4) is still linear in parameters. Let $z_t = (\tilde{y}_{t-1}, 1, \sin(2\pi kt/T), \cos(2\pi kt/T), t)'$, $\theta(\tau) = (\phi, \alpha(\tau), \alpha_k, \beta_k, \gamma)'$,$\alpha(\tau) = \alpha_0 + F_\mu^{-1}(\tau)$, the estimator $\hat{\theta}(\tau)$ could be obtained by solving the following minimization problem:

$$\hat{\theta}(\tau) = \underset{\theta}{argmin} \sum_{t=1}^{T} \rho_\tau(y_t - \theta(\tau)'z_t), \tag{5}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ is the check function.

For asymptotic analysis, we impose the following assumptions first.

Assumption 1. $\{\mu_t\}$ are i.i.d random variables with mean 0 and variance $\sigma^2 < \infty$.

Assumption 2. The distribution of $\{\mu_t\}$, $F(\mu)$, has a continuous density $f(\mu)$ with $f(\mu) > 0$ on $\{\mu: 0 < f(\mu) < 1\}$.

For simplicity and without loss of generality, the error term μ_t is assumed to be i.i.d, thus excluding any linear dependence. However, our analysis could be easily extended to more general case where weak dependence is allowed. Identical distribution assumption is needed for the asymptotic analysis of the QKS test and QCM test and also simplifies the bootstrap procedure. Assumption 2 is quite standard in the literature of quantile regression study, see Koenker and Xiao, 2004.

Since different components of $\hat{\theta}(\tau)$ have different convergence rates, define diagonal matrix $G_T = diag(T, T^{1/2}, T^{1/2}, T^{1/2}, T^{3/2})$. Theorem 1 below gives the asymptotic distribution of $\hat{\theta}(\tau)$.

Theorem 1. *Under Assumptions 1 and 2,*