Finance Research Letters 000 (2017) 1–8

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Efficient estimation of expected stock price returns[☆]

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A R T I C L E I N F O

Article history:
Received 5 January 2017
Revised 5 July 2017
Accepted 12 August 2017
Available online xxx

JEL classification:
G10
G11
G12

Keywords:
Variance gamma model
Digital moment estimation
Self decomposable laws
Limit laws

A B S T R A C T

Daily asset returns are modeled using self decomposable limit laws and the structure is used to estimate the density of the uncentered data. Estimates of mean returns are a byproduct of the density estimate. Estimates of mean returns via density estimation have significantly lower standard errors when compared to estimates derived via the usual method of straight averaging.

1. Introduction

Expected returns of financial stocks are a critical component of many studies in financial markets. Expected returns are typically estimated by averaging daily returns. Black (1993) emphasized that estimating expected returns is of far greater importance than explaining expected returns. He suggested that estimation by averaging will often require very long data records and argued that theory could help circumvent data deficiencies. This paper suggests such a theory.

The intuition regarding the theory is as follows. Suppose there is a good theoretical model of how a random return is generated. This theoretical model may then be coupled with a direct modeling of the uncentered return distribution. The expected return is a byproduct. By leveraging properties of the random variable, the estimation of expected returns can be improved compared to a straight averaging approach based on virtually no data generating theory.

Hence we model uncentered return data and estimate its full distribution. In this distribution there are two means of interest, the expected the log price relative and the expected percentage change. Using the full distribution approach, both expected values are estimated substantially more efficiently as compared to straight averaging.

For financial returns the return distribution may be assumed to be in the class of limit laws representing the effects of many shocks. The class can be thought of as arising from sums of suitably centered and scaled independent shocks. This class was identified and studied by Lévy (1937) and Khintchine (1938) and is known as the self decomposable laws: infinitely divisible laws with a special structure on their Lévy densities or the jump arrival rates as functions of the jump

☆ I acknowledge helpful suggestions and discussions with Piet deJong and King Wang on the subject of this paper. Any remaining errors are solely my responsibility.

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http://dx.doi.org/10.1016/j.frl.2017.08.001
1544-6123/© 2017 Published by Elsevier Inc.

Table 1
Std. Err. ratios log rpice relative.

Quantile	D1/D2	D1/Avg.	D2/Avg.
1	0.8213	0.0696	0.0696
5	0.9850	0.0802	0.0802
10	0.9924	0.0873	0.0873
25	0.9959	0.1041	0.1041
50	0.9978	0.1322	0.1322
75	0.9995	0.1736	0.1736
90	1.0214	0.2234	0.2234
95	1.0656	0.2600	0.2600
99	1.2033	0.3452	0.3452

Table 2
Std. Err. ratios percentage change.

Quantile	D1/D2	D1/Avg.	D2/Avg.
1	0.8261	0.0696	0.0696
5	0.9824	0.0802	0.0802
10	0.9913	0.0872	0.0872
25	0.9959	0.1040	0.1040
50	0.9981	0.1321	0.1321
75	1.0000	0.1735	0.1735
90	1.0220	0.2233	0.2233
95	1.0661	0.2601	0.2601
99	1.2065	0.3454	0.3454

size. Specifically, the arrival rates multiplied by the absolute value of the jump size must be decreasing functions of the absolute jump size (Sato, 1999).

The second aspect about price changes exploited in our estimation is that price moves must be surprises. As a consequence movement times are unpredictable and modeled by Poisson arrival times of moves of different sizes. The analysis is then reduced to the class of pure jump self decomposable laws.

In this subclass there is a particularly simple and tractable model with closed forms for the density and the characteristic function: the variance gamma law (Madan and Seneta, 1990; Madan et al., 1998). The jump arrival rates when multiplied by the absolute value of the jump size are negative exponentials of the absolute jumps size and decreasing functions of the absolute jump size. The variance gamma law is probably one of the simplest self decomposable laws that has three parameters affecting skewness and kurtosis. We therefore estimate means indirectly by first estimating the variance gamma arrival rate function consistent with the uncentered data. This complements earlier results from Madan (2016) where it was shown that centering the data using average mean returns adds noise to the estimation and destroys the quality of fit to the data as compared to directly fitting the uncentered data. In this context centering by averaging can be seen as a misstep.

It may be anticipated that improvements in the underlying distributional assumptions would be expected to enhance the efficiency of expected return estimation along with other aspect of interest. In this regard recent improvements to the variance gamma model delivered by the four parameter bilateral gamma model investigated in Madan and Wang (2017), and Madan et al. (2017) provide such avenues for possible efficiency enhancement. The estimation procedure of matching tail probabilities remains a good estimation procedure, especially if the densities are available in closed form. Further attention to extreme tails may require other developments when this is not the case and observations occur in the extremes. For the bilateral gamma the estimation employed in Madan and Wang (2017) and Madan et al. (2017) employed Fourier inversion for the density.

Better estimates of expected returns are a critical input for numerous financial activities. They include the testing of asset pricing theories, the construction of optimal portfolios from a variety of perspectives with respect to objective functions. Furthermore, improvements in modeling the distribution of returns are critical to a number of issues in risk management, be it the setting of capital reserves or the evaluation of risk exposures. Additional research into the modeling of factor conditional return distributions is then called for and would lead to new models that generate naturally nonlinear factor conditional expected returns. The latter are critical for the implementation of dynamic optimization models.

The outline of the rest of the paper is as follows. The variance gamma model is introduced in Section 2 using two distinct parameterizations. Section 3 presents the estimation strategy employed. Formulas for standard errors are presented in Section 4. Data and estimation results are presented in Section 5. Section 6 concludes.

2. The variance gamma model

Given a positive or negative jump in the log price relative, write $z > 0$ for the absolute value of the jump size. Further let $k(z)$ and $k(-z)$, for $z > 0$ be the arrival rate of jumps of size z and $-z$.