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Comparison of utility indifference pricing and mean-variance approach under normal mixture[☆]

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ABSTRACT

We study utility indifference pricing in order to measure a random cash flow. We evaluate a utility indifference price with an exponential utility function, which we call a risk-sensitive value measure, under the class of normal mixture distributions. It has desirable properties as a value measure. We compare the risk-sensitive value measure and mean-variance approach and provide an empirical application.

1. Introduction

Evaluation of an uncertain project and/or future cash flows is fundamental in finance. Normally we evaluate it to compute (net) present value by discounting expected future or random cash flows with a discounted rate in the finance literature (see, e.g., Bodie and Merton, 2000). However, this way evaluation of its risk is solely dealt with via the discounted rate, which tends to make its risk evaluation ad hoc. Since evaluation of a project and/or random cash flows is fundamental in finance, we consider it is better to have various ways to deal with this important problem. In this paper, we consider an alternative method under which risk of a project and/or uncertain cash flows is more systematically evaluated. In other words, we consider an expected utility function approach with a utility function given by $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$ with $\alpha > 0$ to evaluate a random cash flow $\boldsymbol{X}$ where α denotes degree of risk aversion. Value of a random cash flow $\boldsymbol{X}$ is given by the solution ν of the equation $E[u(-\nu + \boldsymbol{X})] = 0$ where E denotes expectation. We call the solution the utility indifference price (UIP) of $\boldsymbol{X}$, denoted as UIP($\boldsymbol{X}$). When I_0 is an initial cash outflow at time zero, we call the solution ν the utility indifference net price (UINP), denoted as UINP($\boldsymbol{X}$), of the equation $E[u(-\nu - I_0 + \boldsymbol{X})] = 0$. If the utility indifference net price UINP($\boldsymbol{X}$) is positive, then the project or investment plan associated with $\boldsymbol{X}$ should be implemented. If the utility indifference net price UINP($\boldsymbol{X}$) is negative, then the project or investment plan should be discarded. Thus, this decision rule provides an alternative project or investment implementation principle in comparison to the popular net present value (NPV) decision rule. The expected utility function approach has been studied in economics for many years. There is also a book by Carmona (2009) on developments of utility indifference pricing.

Suppose a project has a future cash flow from time 1 to T given by

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$$\boldsymbol{C} = \{C_1, C_2, \cdots, C_T\}$$

where the cash flow $\boldsymbol{C}$ is uncertain and hence random. When an appropriate discount rate is given by r, the present value (PV) of the project is given as follows:

$$PV(\boldsymbol{C}) = \sum_{t=1}^{T} \frac{E[C_t]}{(1+r)^t}.$$

If an initial investment at time 0 is given by I_0, the NPV of the project is given by

$$NPV(\boldsymbol{C}) = -I_0 + PV(C).$$

On the other hand, when the cash flow is random, it is natural to treat its present value as random. We define the random present value (RPV) of $\boldsymbol{C}$, denoted as RPV($\boldsymbol{C}$), as follows:

$$RPV(\boldsymbol{C}) = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}.$$

We call the UIP of RPV($\boldsymbol{C}$) the utility indifference present value (UIPV), denoted as UIPV($\boldsymbol{C}$), which is given by the solution ν of the following equation

$$E[u(-\nu + RPV(C))] = 0.$$

Similarly we call the utility indifference net present value (UINPV), denoted as UINPV($\boldsymbol{C}$), UINP(RPV($\boldsymbol{C}$)), i.e., the solution $\hat{\nu}$ of the following equation given by

$$E[u(-\hat{\nu} - I_0 + RPV(C))] = 0.$$

There exist many studies of risk measures and extreme events to evaluate risk. To the best of our knowledge, however, it seems there are not many studies of value measures to value random cash flows besides the mean-variance (MV) approach and utility indifference pricing. However, utility indifference pricing has been used mainly in theoretical works in continuous time processes such as diffusion processes and jump processes with theoretical analytical results which are difficult to verify empirically (see, e.g., chapters and references in Carmona, 2009). Our approach is presented basically in a static setting in its current presentation, i.e., in a single period model, so that it is valid when the underlying cash flow is independently and identically distributed. Validity in other setting is beyond the scope of this paper. Our approach is unique as it is given in discrete time and contains a simulation experiment and an empirical example.

Miyahara (2010) proposed the expected utility function approach described above to evaluate a random cash flow $\boldsymbol{X}$ or a RPV($\boldsymbol{C}$) and proved UIP($\boldsymbol{X}$) (UINP($\boldsymbol{X}$)) or UIPV($\boldsymbol{C}$) (UINPV($\boldsymbol{C}$)) satisfies several desirable properties an evaluation function of a project or random cash flow ought to satisfy and thus justified UIP (UINP) or UIPV (UINPV) as an evaluation function of a random cash flow. The UIP($\boldsymbol{X}$) can be easily seen to be given by

$$-\frac{1}{\alpha}\ln E[e^{-\alpha \boldsymbol{X}}]$$

when the utility function is given by $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$ and Miyahara (2010) called it a risk-sensitive value measure (RSVM) of $\boldsymbol{X}$ because it responds sensitively to underlying risk. Therefore, when we use the above exponential utility function, we can obtain its UIP and UIPV explicitly, which makes its computational task easy. Furthermore it is shown (given as Proposition 2 in the next section) that the exponential utility function $u(x) = \frac{1}{\alpha}(1 - e^{-\alpha x})$ and the RSVM is the only utility function and the only UIP among $\boldsymbol{C}^2$-class of utility functions under a certain condition (see the condition of Proposition 2 in the next section). Since the UIP is a concave monetary value measure, which is given as Proposition 1 in the next section, the RSVM, which is the UIP with the exponential utility function, is the only UIP that is a concave monetary value measure among UIPs with $\boldsymbol{C}^2$-class of utility functions under the condition of Proposition 2. The RSVM is thus justified. In order to fulfill Miyahara's expected utility function approach, the remaining task is to evaluate expectation in the RSVM of $\boldsymbol{X}$ and $RPV(\boldsymbol{C})$, i.e., expectation in $E[e^{-\alpha \boldsymbol{X}}]$ and $E[e^{-\alpha RPV(\boldsymbol{C})}]$ for a given degree of risk aversion α. In this paper we intend to carry out the remaining task, i.e., to evaluate expectation in $E[e^{-\alpha \boldsymbol{X}}]$ when $\boldsymbol{X}$ follows the class of discrete normal mixture distributions.

It is well known that the class of discrete normal mixture distributions is flexible enough to capture various characteristics which indicate not only symmetric distributions but also leptokurtic, skewed, and multimodal distributions often observed in financial instruments (cf., e.g., Everitt and Hand, 1981; Titterington et al., 1985; Mc Lachlan and Peel, 2000; Kon, 1984; Ritchey, 1990; Chin et al., 1999; Brigo and Mercurio, 2001, and Alexander, 2004). A beautiful thing about the class of discrete normal mixture distributions is that once a discrete normal mixture distribution is estimated, the RSVM can be immediately derived by plugging its estimated parameters into the moment-generating function (MGF) of the discrete normal mixture distribution since $E[e^{-\alpha \boldsymbol{X}}]$ in the RSVM of $\boldsymbol{X}$ is, besides the minus sign, nothing but the MGF of $\boldsymbol{X}$. We remark the MGF of a discrete normal mixture distribution can be easily derived from that of each component. On the other hand, estimation of discrete normal mixture distributions has been studied for several decades in statistics and econometrics literature (cf., e.g., Everitt and Hand, 1981; Titterington et al., 1985, and Mc Lachlan and Peel, 2000). We can make use of appropriate methods in the literature to estimate them. There also exist many financial applications of discrete normal mixture distributions. See, e.g., Kon (1984), Ritchey (1990), Chin et al. (1999), Brigo and Mercurio (2001), and Alexander (2004) among others. Therefore it is not unnatural to consider the underlying data-generating