Finance Research Letters 000 (2017) 1–8

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Long vs. short term asymmetry in volatility and the term structure of risk

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ARTICLE INFO

Article history:
Received 12 February 2017
Revised 21 May 2017
Accepted 7 June 2017
Available online xxx

JEL Classification:
C22
C51
C58
G17
G15

Keywords:
Financial econometrics
GARCH
Memory
Risk prediction
Skewness

ABSTRACT

This short paper introduces the distinction between short and long term asymmetric effects in volatilities. With short term asymmetry we refer to the conventional one, i.e. the asymmetric response of current volatility to the most recent return shocks. In addition, we argue that there may be asymmetries with respect to the way the effect of past return shocks propagate over time. We refer to this as long term asymmetry and propose a model that enables the study of the potential occurrence of such a feature. In an empirical application using stock market index data we find evidence of the joint presence of short and long term asymmetric effects and demonstrate important implications for risk predictions. In particular, positive return shocks is ascribed substantial significance for long term risk prediction.

1. Introduction

Understanding the dynamics of financial return volatility is crucially important in financial contexts such as risk management and portfolio selection. To this end the ARCH/GARCH framework of Engle (1982) and Bollerslev (1986) stands out as the single most important tool and since the birth of the basic models the literature has exploded with different extensions (see Andersen et al., 2006; Bollerslev, 2008). Seemingly, the most popular and empirically relevant ones are those attempting to cope with the stylized fact of asymmetry. This property is most notable for equity returns and it refers to the fact that return volatility tends to rise more following negative return shocks than positive ones. One of the most popular model (see McAleer, 2014, for a recent discussion) of how to cope with it appears to be the asymmetric GARCH (GJR-GARCH) of Glosten et al. (1993). Other commonly employed alternatives include the exponential GARCH model of Nelson (1990). A more recent extension is the dynamic asymmetric GARCH (DAGARCH) of Caporin and McAleer (2006) that generalizes the GJR-GARCH to include multiple and time-varying thresholds.

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[1] The financial support from the Browald and Wallander–Hedelius Foundations under grant number W2009-0406:1 is gratefully acknowledged.

http://dx.doi.org/10.1016/j.frl.2017.06.011
1544-6123/

Up til now the effort in terms of modeling has focused on how to best capture the response of current volatility to the most recent return shocks. We refer to this as short term asymmetry. However, given the different role positive and negative returns plays for volatility it does not appear too far fetched to expect asymmetries in the way the effect of past return shocks propagate over time as well. We refer to this as long term asymmetry and propose a simple extension to the basic GJR-GARCH to enable the study of the possible occurrence of such a feature.

Furthermore, an implication of asymmetric effects in one-period volatilities is skewness in multi-period returns. In a recent paper Engle (2011) builds on Guidolin and Timmermann (2006) and focuses on the role of skewness for long term risk predictions. A message from the study is that common volatility models allowing for (short term) asymmetric effects generate to little skewness in the short term and to much skewness in longer term. Thus, an interesting hypothesis is that the further relaxing accomplished by the proposed model gives a more adequate representation of the term structure of skewness and, consequently, the term structure of risk. Indeed, as emphasized in Engle (2009) the financial crises made it very clear that danger may be lurking in the future even though things may seem calm for the moment. Set to reflect short term risks existing risk management systems failed to warn for what was coming ahead.

The paper proceeds as follows. Section 2 outlines the proposed extension to the basic GJR-GARCH. Section 3 gives an account of the term structure of risk and the prediction of it. In Section 4 we provide some empirical results for the S&P 500. Section 5 concludes and discusses further research.

2. Long vs. short asymmetry

We define a return shock process $\{u_t\}$ that is generated in discrete time by

$$u_t = \sqrt{h_t}\varepsilon_t, \tag{1}$$

where $\{\varepsilon_t\} \sim iid(0, 1)$. Returns are given by $y_t = \mu_t + u_t$ and with $\mathcal{F}_t$ denoting the history up to and including time t we have the conditional mean $\mu_t = E(r_t|\mathcal{F}_{t-1})$ and variance $h_t = V(u_t|\mathcal{F}_{t-1}) = V(r_t|\mathcal{F}_{t-1})$, respectively. To allow for asymmetric effects in the specification of h_t we define $u_t^{2+} = h_t\varepsilon_t^{2+} = h_t\varepsilon_t^2\mathbf{1}(\varepsilon_t \geq 0)$ and $u_t^{2-} = h_t\varepsilon_t^{2-} = h_t\varepsilon_t^2\mathbf{1}(\varepsilon_t < 0)$, where $\mathbf{1}(\,\cdot\,)$ is the indicator function. The basic GJR-GARCH specification for the conditional variance may then be defined as

$$h_t = \omega + \alpha^+ u_{t-1}^{2+} + \alpha^- u_{t-1}^{2-} + \beta h_{t-1}. \tag{2}$$

To guarantee a positive variance at all times we require that $\omega > 0$ and $\alpha^+, \alpha^-, \beta \geq 0$. Ling and McAleer (2002) establish conditions for stationarity and ergodicity and the existence of moments for a family of GARCH models including the GJR-GARCH. In this case we add the restriction $\alpha^+ E(\varepsilon^{2+}) + \alpha^- E(\varepsilon^{2-}) + \beta < 1$ for a non-explosive behavior. We say that the model is asymmetric in the short term sense but not in the long term sense since the rate of decay of the effect of past positive and negative return shocks is the same. The difference merely occurs as a result of $\alpha^+ \neq \alpha^-$. This is most easily seen from the corresponding infinite ARCH representation of (2)

$$\begin{aligned} h_t &= \frac{\omega}{1-\beta} + \frac{\alpha^+}{1-\beta L}u_{t-1}^{2+} + \frac{\alpha^-}{1-\beta L}u_{t-1}^{2-} \\ &= \omega^* + \alpha^+(1 + \beta L + (\beta L)^2 + \ldots)u_{t-1}^{2+} + \alpha^-(1 + \beta L + (\beta L)^2 + \ldots)u_{t-1}^{2-}, \end{aligned}$$

where $\omega^* = \omega/(1-\beta)$ and L is the lag operator, i.e. $Lx_t = x_{t-1}$. Now, to parsimoniously accommodate asymmetry also in the long term sense our simple idea is to extend the model to have one β for positive return shocks and one for negative ones, i.e.

$$h_t = \omega^* + \frac{\alpha^+}{1-\beta^+ L}u_{t-1}^{2+} + \frac{\alpha^-}{1-\beta^- L}u_{t-1}^{2-}. \tag{3}$$

We say that there are short term asymmetric effects when $\alpha^+ \neq \alpha^-$, while there is long term asymmetry when $\beta^+ \neq \beta^-$. Obviously, the cases can occur simultaneously. Note that upon multiplying both sides of Eq. (3) by $(1-\beta^+ L)(1-\beta^- L)$ and re-arranging we obtain

$$h_t = \omega + \alpha^+ u_{t-1}^{2+} + \alpha^- u_{t-1}^{2-} - \alpha^+\beta^- u_{t-2}^{2+} - \alpha^-\beta^+ u_{t-2}^{2-} + (\beta^+ + \beta^-)h_{t-1} - \beta^+\beta^- h_{t-2} \tag{4}$$

Thus, our proposed specification may be viewed as a restricted version[2] of a GJR-GARCH(2,2). As such, it does not constitute a new model per se but should rather be viewed as a well-motivated intermediate case between the often used GJR-GARCH(1,1) and the (from a practical perspective) less feasible unrestricted GJR-GARCH(2,2). We expect it to have ergodic and stationary parameter combinations. It as beyond the scope of the present paper to work out the exact conditions though. Here, we considered a single threshold set at zero and a relatively simple lag structure. Being a first development in this direction this seems to be a natural choice. However, a richer lag structure as well as extensions towards multiple and even time-varying thresholds as in Caporin and McAleer (2006) are in principle possible.

[2] The restrictions $\alpha_2^+ = -\alpha_1^+\beta_2$ and $\alpha_2^- = -\alpha_1^-\beta_1$ on the parameters in $h_t = \omega + \alpha_1^+ u_{t-1}^{2+} + \alpha_2^+ u_{t-1}^{2+} + \alpha_1^- u_{t-1}^{2+} + \alpha_2 u_{t-1}^{2+} + \beta_1 + \beta_2)h_{t-1} - \beta_1\beta_2 h_{t-2}$ gives our case.

Please cite this article as: C. Lönnbark, Long vs. short term asymmetry in volatility and the term structure of risk, Finance Research Letters (2017), http://dx.doi.org/10.1016/j.frl.2017.06.011