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Time horizons, lattice structures, and welfare in multi-period matching markets [☆]

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ABSTRACT

We analyze a T-period, two-sided, one-to-one matching market without monetary transfers. Under natural restrictions on agents' preferences, which accommodate switching costs, status-quo bias, and other forms of inter-temporal complementarity, dynamically stable matchings exist. We propose a new ordering of the stable set ensuring that it forms a lattice. We investigate the robustness of dynamically stable matchings with respect to the market's time horizon and frequency of rematching opportunities.

The passage of time is an inescapable dimension of most economic and social interactions. Workers pursue decades-long careers. Students spend years learning at a succession of institutions. Wedding anniversaries are celebrated (and sometimes forgotten). It is therefore surprising that time is absent from typical studies of two-sided matching markets, as originally formulated by Gale and Shapley (1962). In such a market, agents are partitioned into two groups, such as firms and workers, schools and students, or men and women. They must match together to realize benefits. A stable matching results when no agent or pair can deviate to an alternative, mutually-preferable arrangement.

Accounting for the market's time dimension focuses attention on several real-world complications that do not arise when interactions are one time or ephemeral. First, the passage of time allows agents to change their partners.[1] Change

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[1] Change occurs in all standard applications of two-sided matching theory, including marriage/inter-personal relations, employment, and education. Approximately 54 percent of couples married between 1975 and 1979 celebrated their 25th anniversary (Kreider and Ellis, 2011, Table 4) and around a quarter of those born from 1940–1949 were married two or more times by age 55 (Kreider and Ellis, 2011, Table 3). Change occurs even in markets that are more limited in time and place. Bearman et al. (2004) study romantic relationships among high schools students. The authors document multiple relationships among a large fraction of students during their sample period. Similarly, longitudinal evidence from the Bureau of Labor Statistics (2012) notes that the average person born between 1957 and 1964 held 11.3 jobs between the ages of 18 and 46. Finally, there is considerable "churn" among students enrolled in schools. Approximately ten percent of students changed schools in Massachusetts in 2008–2009 (O'Donnell and Gazos, 2010, Table 1). During the same time period, the churn rate in Boston public schools was 25.3 percent (O'Donnell and Gazos, 2010, Table 3).

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introduces a degree of complexity not encountered in the single-period, one-shot case. The order of interactions matters and inter-temporal tradeoffs play a prominent role. Second, decision making over a time horizon is difficult. Habit formation and path-dependence are significant behavioral features. Finally, commitment is not assured; dynamic incentives matter.

This paper examines a T-period, two-sided, one-to-one matching market where agents match with a possibly changing sequence of counter-parties. Two questions guide our investigation:

(i) What are the welfare properties of "stable" market outcomes?
(ii) How sensitive are stable outcomes to the market's time horizon or the frequency of rematching opportunities?

As explained below, our answers to these questions shed light on the properties of dynamic economies, establish connections with familiar static environments, and are suggestive of practical applications.

Our analysis extends the model of Kadam and Kotowski (forthcoming), who define and analyze "dynamically stable" matchings in a multi-period, two-sided matching market. Generalizing the classic notion of stability, dynamically stable matchings, roughly speaking, are sequences of assignments that no individual or pair can "block" at any moment in time.[2] Agents can block an outcome if they can formulate a superior continuation assignment plan independently of others' behavior. Kadam and Kotowski (forthcoming) analyze this solution in a two-period model.[3] They introduce restrictions on agents' preferences, which are defined over sequences of assignments, that ensure a dynamically stable matching exists. An extension of their model considers imperfect information and learning.

We adopt the same baseline model as Kadam and Kotowski (forthcoming), but our analysis differs in thematic focus and technical detail. Foremost, given our interest in the market's time horizon and rematching frequency, our analysis is set in a T-period setting. We identify a tractable class of non-time-separable preferences that guarantee the existence of a dynamically stable matching. This class accommodates common behavioral characteristics, including status quo bias and switching costs (Samuelson and Zeckhauser, 1988). As we show, and despite their connotation, these wrinkles need not imply a stable matching's immutability.

To understand our model's welfare implications, we investigate the lattice structure of the set of dynamically stable matchings. A lattice is a partially-ordered set where any two elements have a unique supremum and a unique infimum (Birkhoff, 1940). In a one-period setting, the set of stable matchings forms a lattice when ordered by the "common preference" (defined below) of agents on one side of the market.[4] Thus, any emergent systematic structure is indicative of welfare and distributional outcomes. The lattice of stable matchings has been investigated in an array of closely-related static models (Knuth, 1976; Blair, 1984, 1988; Roth, 1984, 1985b; Sotomayor, 1999; Alkan, 2001, 2002). Its applicability and characteristics in dynamic markets is less well-understood.

Our analysis yields subtle conclusions and qualifications. First, despite a structured preference domain, we document several important differences between multi-period and one-period markets. Dynamically stable matchings need not be Pareto optimal, due to match mis-coordination. Moreover, man- or woman-optimal stable matchings do not always exist.[5] These are focal outcomes in one-period economies due to their coincidence with assignments identified by Gale and Shapley's (1962) deferred acceptance algorithm. As a counterpoint to these observations, we propose a new ordering of the stable set ensuring that it forms a lattice. Our ordering is also derived from agents' preferences, thereby maintaining its welfare interpretation.

As our model emphasizes multi-period interactions, it is also important to understand how the set of stable outcomes changes with the market's time horizon, the parameter T in our notation. Equivalently, this parameter defines the number of rematching opportunities over an otherwise fixed time period. Its determination often depends on the analyst's discretion. Thus, to gauge robustness, it is useful to know how stable matchings relate to one another for nearby values of T. We show that intuitive projections and embeddings of stable outcomes are possible as T changes. Such conclusions are not immediate since, for instance, adjustment of T changes the number of blocking opportunities.

Contributions and applications Our analysis develops tools for the analysis of complex two-sided matching economies and is suggestive of applications. Foremost, we further the development of dynamic stability as a simple and intuitive benchmark for understanding multi-period interactions in cooperative matching models. As part of this analysis, we introduce a tractable and behaviorally-plausible class of preferences capturing common characteristics of inter-temporal decision making, including status quo bias and switching costs. By probing the stable set's lattice structure, we identify assumptions that allow familiar intuitions concerning welfare and optimality from one-period economies to carry over to our richer setting. The resulting model is indicative of interactions seen in many real-world markets where matching and long-run incentives

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[2] There exist many proposed generalizations of Gale and Shapley's (1962) stability criterion to a dynamic setting. We discuss the related literature in Section 1.

[3] In an appendix to their paper, Kadam and Kotowski (forthcoming) explain how some of their results can be extended to a T-period setting. The analysis herein does not draw on those generalizations directly. A concurrent contribution of our analysis is the identification of restrictions that simplify the study of T-period markets.

[4] This observation is attributed to Conway by Knuth (1976).

[5] A stable matching is man-optimal if each man prefers his assignment in that matching to his assignment in all other stable matchings. A woman-optimal stable matching is defined analogously.