Games and Economic Behavior 109 (2018) 82–98

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

A study of the nucleolus in the nested cost-sharing problem: Axiomatic and strategic perspectives ☆

Cheng-Cheng Hu [a], Min-Hung Tsay [b], Chun-Hsien Yeh [c,*]

[a] *Department of Economics, National Cheng Kung University, Tainan 701, Taiwan*
[b] *Department of Economics, National Chung Cheng University, Chia-Yi 621, Taiwan*
[c] *Institute of Economics, Academia Sinica, Taipei 115, Taiwan*

ARTICLE INFO

Article history:
Received 17 February 2015
Available online 20 December 2017

JEL classification:
C71
C72
D63

Keywords:
Consistency
Nash program
Axiomatization
Strategic justification
Nucleolus
Airport problem

ABSTRACT

We investigate the nucleolus from both axiomatic and strategic perspectives in the nested cost-sharing problem in which the cost of a public facility has to be shared among agents having different needs for it. We adopt a Right-endpoint Subtraction (RS) formulation, which underlies these two properties: *RS bilateral consistency* and *RS converse consistency*. As we show, the nucleolus is the only *RS bilaterally consistent* (or *RS conversely consistent*) rule satisfying *equal treatment of equals* and *last-agent cost additivity*. In addition, we introduce a game exploiting the two properties to strategically justify the nucleolus. Our results, together with the axiomatization and strategic justification of the Constrained Equal Benefits (CEB) rule in Hu et al. (2012), show that adopting different formulations to define a reduced problem leads to axiomatizing and, in particular, strategically justifying the CEB rule and the nucleolus in the nested cost-sharing problem.

1. Introduction

The nucleolus (Schmeidler, 1969) is an important solution concept for Transferable Utility (TU) games. An allocation chosen by it minimizes the dissatisfactions of coalitions in the lexicographic order beginning with the worst-treated coalition. Our purpose is to axiomatize and strategically justify the nucleolus in the following "nested cost-sharing problem", which is "dual" to the auction game (Graham et al., 1990) and has many real life applications.[1] Several agents jointly use a public facility and have different needs for it. If the facility can satisfy the need of an agent, then it can also satisfy any smaller

☆ We would like to thank William Thomson, Youngsub Chun, Hans Peters, René van den Brink, Yuan Ju and Biung-Ghi Ju for their helpful comments. In particular, we are grateful to the associate editor in charge and two anonymous referees for their constructive comments. Tsay acknowledges financial support from the grant of Ministry of Science and Technology, Taiwan (MOST-105-2410-H-194-011-MY2). Yeh acknowledges financial support from Career Development Award of Academia Sinica, Taiwan (AS-99-CDA-H01) and the grant of Ministry of Science and Technology, Taiwan (MOST-105-2410-H-001-002-MY3). We are responsible for any remaining deficiencies.

* Corresponding author.
E-mail addresses: hucc@mail.ncku.edu.tw (C.-C. Hu), mhtsay@ccu.edu.tw (M.-H. Tsay), chyeh@econ.sinica.edu.tw (C.-H. Yeh).

[1] An example is the "airport problem". Several airlines jointly use a runway. Different airlines need runways of different lengths. The bigger the plane an airline operates, the longer the runway it needs. A runway serving a given plane can also serve any smaller plane at no extra cost. To accommodate all airlines, the length of the runway must be long enough for the biggest plane any airline operates. How should the maintenance cost of the runway be shared among the airlines? Other examples are the "irrigation problem", the "taxi fare sharing problem", and the "highway user fee problem". The first study of this kind was Littlechild and Owen (1973). For a survey, see Thomson (2014).

https://doi.org/10.1016/j.geb.2017.11.012
0899-8256/

need at no extra cost. Thus, the facility should be provided so as to satisfy the largest need of the agents. How should the cost of making or maintaining the facility be shared among the agents? A "rule" is a function that associates with each such problem an allocation of the cost, called a "contributions vector".

Our analysis closely relates to two well-known principles: "bilateral consistency" and "converse consistency". Consider a problem and an allocation chosen for it. Imagine that all agents, except for two agents, leave with their "components of the allocation". We derive a "two-agent reduced problem" by reevaluating the situation from the viewpoint of the remaining two agents. Bilateral consistency says that the restriction of the allocation to this two-agent subgroup should be chosen for the associated two-agent reduced problem. Converse consistency says that an allocation should be chosen for some problem if, for each two-agent reduced problem associated with it, its restriction to this subgroup is chosen.[2]

Since the cost structure of the problem is nested (see Fig. 1, for an example), different agents use the facility differently, which makes different formulations of a reduced problem available. Potters and Sudhölter (1999) propose Right-endpoint Subtraction (RS) and Left-endpoint Subtraction (LS) formulations for the nested cost-sharing problem.[3] We adopt their RS formulation underlying "RS bilateral consistency" and "RS converse consistency"; in contrast, Hu et al. (2012) adopt the LS formulation underlying "LS bilateral consistency" and "LS converse consistency". Hu et al. (2012) show that the "Constrained Equal Benefits (CEB) rule"[4] is the only *LS bilaterally consistent* (or *LS conversely consistent*) rule satisfying "equal treatment of equals" and "last-agent cost additivity". We show that axiomatizations of the nucleolus are obtained by replacing *LS bilateral consistency* with *RS bilateral consistency* (and *LS converse consistency* with *RS converse consistency*) in their axiomatizations of the CEB rule. These results should remind us of how sensitive axiomatic analysis is to the manner in which reduced problems are specified.

In light of the above axiomatizations, we ask whether adopting different formulations of a reduced problem could lead to strategic justification of different rules.[5] To tackle this question, we revise the game in Hu et al. (2012) based on *RS bilateral consistency* and on *RS converse consistency*.[6] The resulting game introduced below is a three-stage extensive form game whose construction is in line with Davis and Maschler (1965)'s consistency property for TU games. Particularly, in Stage 3, agents are given the opportunity to minimize their contributions in two-agent reduced problems by picking a coalition to work with.[7] Let an agent with the largest need be the responder, and let all the other agents be proposers.

***Stage 1*:** Each proposer proposes her own contribution to the total cost (the cost of satisfying the largest need among all agents).

***Stage 2*:** The responder either accepts their proposed contributions, in which case she contributes the residual cost, or she rejects them. In the case of a rejection, she takes one agent, called partner, to re-negotiate their contributions. All the others contribute the amounts they proposed.

***Stage 3*:** A fair coin chooses one agent between the responder and the partner. The chosen agent is given the opportunity to minimize her contribution by (i) selecting her teammates from all the other agents, and (ii) using the total contribution proposed by her teammates to cover the cost of satisfying the largest need among her and her teammates. The other agent then uses the remaining contributions to cover the residual cost.

We show that for each such problem, there is a unique Subgame Perfect Equilibrium (SPE) outcome of our game and, moreover, it is the nucleolus contributions vector.

Our paper makes the following contributions. First, since we consider a class of cost-sharing problems, it is natural that each agent is required to propose the amount she wishes to contribute. Second, compared with the existing strategic justifications of the nucleolus, our result is neither dependent on any particular order of agents nor does it invoke any particular rule in specifying agents' payoffs. Third, in the existing literature, it is typical to either axiomatize or strategically justify a rule (or a class of rules). In contrast, we not only axiomatize and strategically justify the nucleolus but also offer axiomatic and, in particular, strategic comparisons between the nucleolus and the CEB rule in the nested cost-sharing problem.

Related literature on the study of strategic justification of the nucleolus, Serrano (1993) offers a three-agent strategic justification of the nucleolus for TU games and shows that it is impossible to extend his result to more than three agents. Since then, attention has been drawn to smaller domains. For instance, Serrano (1995) offers a strategic justification of the nucleolus for "bankruptcy problems". Arin et al. (2009) and our paper provide strategic justifications of the nucleolus for the model under consideration.

[2] The two principles have been applied to several contexts such as taxation, bargaining, and social choice. For references, see Peleg (1986) and Yeh (2006). For a survey on the principles, see Thomson (2010).

[3] Potters and Sudhölter (1999) call the reduced problem derived from the RS (LS) formulation the "ν-(ψ-)reduced airport problem".

[4] The rule makes agents' benefits (defined as the difference between an agent's cost and her contribution) equal, subject to no one paying a negative amount.

[5] Nash (1953) initiates the study of cooperative solution concepts from the strategic perspective, which is now called the Nash program. For references, see Gul (1989), Hart and Mas-Colell (1996), Perry and Reny (1994), Perez-Castrillo and Wettstein (2001), van den Brink et al. (2013), and Ju et al. (2014). For a survey, see Serrano (2005).

[6] The consistency property of a rule plays an important role in strategically justifying the rule (Krishna and Serrano, 1996). For this line of research, see Serrano (1997), Dagan et al. (1997), and Chang and Hu (2008).

[7] A similar design can be found in Chang and Hu (2017) for their strategic justification of a solution concept, the "kernel", for TU games.