Games and Economic Behavior ••• (••••) •••–•••

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Games and Economic Behavior

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Values for environments with externalities – The average approach [☆]

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ARTICLE INFO

Article history:
Received 7 July 2016
Available online xxxx

JEL classification:
D62
C71

Keywords:
Externalities
Sharing the surplus
Average approach

ABSTRACT

We propose the "average approach," where the worth of a coalition is a weighted average of its worth for different partitions of the players' set, as a unifying method to extend values for characteristic function form games. Our method allows us to extend the equal division value, the equal surplus value, the consensus value, the λ-egalitarian Shapley value, and the family of least-square values. For each of the first three extensions, we also provide an axiomatic characterization of a particular value for partition function form games. And for each of the last two extensions, we find a family of values that satisfy the properties.
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1. Introduction

A central question in game theory is how players can share the gains from cooperation. Shapley (1953) addresses this issue for cooperative games in characteristic function form, where the description of a game specifies the resources every group of players has available for distribution among its members. He proposes the use of a sharing rule, or a value (known as the "Shapley value"), that satisfies the axioms of symmetry, carrier (which amounts to the efficiency plus dummy player axioms), and additivity. The Shapley value has been studied, interpreted, and characterized in many different ways.[1] Interestingly, it arises from apparently distinct and unrelated approaches (axiomatic, marginalistic, potential, dividends). In addition, it has been extremely influential in later proposals for surplus sharing. Many researchers have followed the path Lloyd Shapley laid out and put forward some modifications of the Shapley axioms to define new values for sharing the surplus generated through cooperation.

A shortcoming of describing a cooperative environment through a characteristic function form game is that it disregards the possible existence of externalities among groups. Externalities in economic or political environments are the norm rather than the exception. For instance, research joint ventures, mergers and acquisitions, international negotiations on environmental issues, and trade agreements all exhibit important cross effects, namely, the gain that a group of agents

☆ We would like to thank Ezra Einy, Ori Haimanko, an anonymous reviewer, as well as participants at the 27th International Conference on Game Theory in Stony Brook for helpful comments. We gratefully acknowledge financial support from the Ministerio de Economía y Competitividad and Feder (ECO2015-63679-P), Generalitat de Catalunya (2014SGR-142, and ICREA Academia), and Severo Ochoa Programme (SEV-2015-0563). The first two authors are fellows of CESifo and MOVE.
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[1] See, for instance, Roth (1988).

http://dx.doi.org/10.1016/j.geb.2017.08.003
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obtains depends on the groups formed by the other players. A formal description of such settings with externalities is given by Thrall and Lucas (1963) who introduce games in partition function form.

To allow value theory to address environments with externalities, several papers adapt and at times augment the Shapley axioms of efficiency, symmetry, linearity, and dummy player to partition function form games. This leads to several new sharing methods for environments with externalities (see, e.g., Myerson, 1977; Bolger, 1989; Albizuri et al., 2005; Macho-Stadler et al., 2007; Pham Do and Norde, 2007; and McQuillin, 2009).[2]

In this paper, we suggest a unifying method, the "average approach," of extending any value for characteristic function form games that satisfies the above axioms of efficiency, symmetry, and linearity to partition function form games. The average approach associates to each group of players a worth that is some weighted average of what the group can obtain for all possible partitions of the other players. This yields a game with no externalities, the value of which determines the value for the original game. The axiomatic basis for this procedure is given by a natural extension of the symmetry axiom for partition form games, the "strong symmetry axiom," introduced by Macho-Stadler et al. (2007) (MPW, 2007, hereafter). The strong symmetry axiom captures the idea that all players with identical influence in a game should receive the same outcome. It is also related to the property of equal treatment of equals.

We use this approach to propose extensions of several well-known values as well as families of values defined for games without externalities. In addition, we suggest generalizations of the axioms proposed for characteristic function form games (such as nullifying player, neutral dummy player, or coalitional monotonicity), to adapt them to situations with externalities. Our method allows us to extend the *equal division value* (Van den Brink, 2007), the *equal surplus value* (Driessen and Funaki, 1991), the *consensus value* (Ju et al., 2007), the λ-*egalitarian Shapley value* (Joosten, 1996), and the *family of least-square values* (Ruiz et al., 1998). For each of the first three extensions, we also provide an axiomatic characterization of a particular value for partition function form games. It is worth noting that the extension of the consensus value through the average approach coincides with the one proposed by Ju (2007).[3] For each of the last two extensions, a family of values that satisfy the properties is found.

The paper proceeds as follows. In section 2, we present the environment. Section 3 introduces the basic axioms and the average approach. Sections 4 and 5 present the extensions of several values and families of values. Finally, section 6 briefly concludes.

2. The environment

The environment we study can be described as follows. We denote by $N = \{1, ..., n\}$ the set of players. A coalition S is a group of s players, that is, a subset of N, $S \subseteq N$. An embedded coalition specifies the coalition as well as the structure of coalitions formed by the other players, that is, an embedded coalition is a pair (S, P), where S is a coalition and $P \ni S$ is a partition of N. Let $\mathcal{P}$ be the set of all partitions of N. We use the convention that the empty set $\varnothing$ is in P for every $P \in \mathcal{P}$. We denote by $\mathcal{P}_S = \{P \in \mathcal{P} \mid S \in P\}$ the set of partitions including S. Then, the set of embedded coalitions, ECL, is defined as:

$$ECL = \{(S, P) \mid P \in \mathcal{P}_S\}.$$

Also, we denote by $[S]$ the partition of S consisting of all the singleton players in S, that is, $[S] = \left\{\{i\}_{i \in S}\right\}$.

We denote by v a game in *partition function form* (PFF), where $v : ECL \to \mathbb{R}$ is a function that associates a real number with each embedded coalition (S, P). We assume that the function satisfies $v(\varnothing, P) = 0$ for all $P \in \mathcal{P}$. Let $\mathcal{G}^N$ be the set of games in PFF with players in N. We interpret $v(S, P)$ as the worth of coalition S when the players are organized according to the partition P. In our set up, players in a coalition can make transfers among them, that is, we consider transferable utility (TU) games. The worth $v(S, P)$ may depend on the partition P. This implies that the organization of the players outside S may create a positive or negative externality on the payoff that players in a coalition S can jointly obtain.

An important class of games with externalities that forms a basis for the set $\mathcal{G}^N$ is given by the games $w_{S,P}$ defined by

$$w_{S,P}(S, P) = w_{S,P}(N, (N, \varnothing)) = 1, \text{ and } w_{S,P}(S', P') = 0 \text{ otherwise.}$$

In the game $w_{S,P} \in \mathcal{G}^N$ there are only two scenarios where a coalition has a positive worth, the first is for the coalition S when the players are organized according to the partition P, and the second is for the grand coalition. We will refer to this class of games as the "canonical basis" for $\mathcal{G}^N$.

Some games in $\mathcal{G}^N$ do not have externalities. A game is without externalities if the worth of any coalition S is independent of the way the other players are organized. A game without externalities satisfies $v(S, P) = v(S, P')$ for any $P, P' \in \mathcal{P}_S$ and any coalition $S \subseteq N$. We denote such a game by $\hat{v}$. Since the worth of a coalition S in the game $\hat{v}$ can be written without reference to the organization of the remaining players, we can write $\hat{v}(S) \equiv \hat{v}(S, P)$ for all $P \in \mathcal{P}_S$ and all $S \subseteq N$. We denote by G^N the set of games without externalities with players in N, which corresponds to the set of TU games in *characteristic function form* (CFF).

[2] Different methods to extend the Shapley value are proposed by de Clippel and Serrano (2008), who rely on the marginal approach, and by Dutta et al. (2010), who use the potential approach.
[3] Besides the previous papers, Hernández-Lamoneda et al. (2009) extend the solidarity value (Nowak and Radzik, 1994) to games with externalities.

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