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# Bounded memory and incomplete information

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ABSTRACT

This paper studies incomplete information games where players observe only a summary statistic of the history, including reputation games as a special case. A recursive characterization of the equilibrium payoff set is derived for the case where time is observable, relating it to a self-generating set of tuples that capture equilibrium behavior and payoffs. With unobservable time, equilibria have a particularly simple interpretation as self-generating points. The tools are applied to a product choice game where the firm may be an "honest" commitment type and consumers have 1-period memory with imperfect monitoring, solving for the worst equilibrium payoff. The recursive algorithm shows that the observable-time game allows lower equilibrium payoffs due to non-stationary behavior.


## 1. Introduction

Models with incomplete information, such as reputation games, usually assume that the full history of past behavior is observed, yet in many real world settings the view into the past is limited. For example, events on credit histories are deleted after a certain time in many countries, and workers typically provide only recent references to prospective employers.[2] Many online markets display only recent reviews of sellers or make older ones less prominent (e.g. eBay) and unlikely to be seen by buyers, or display star ratings on Amazon or Yelp. In 2014, the Court of Justice of the European Union ruled in *Google v. Costeja* that individuals have a "right to be forgotten" and may demand search engines remove links to old information "in light of the time that has elapsed." Under incomplete information, what happens when agents know the record of today's behavior will eventually be restricted or forgotten?

The first half of this paper introduces general tools to study incomplete information environments where players observe only a summary statistic of past events from a finite set, referred to here as *bounded memory*. The main result is a recursive characterization of the equilibrium payoff set for this general class of games, extending Doraszelski and Escobar's (2012) (hereafter DE) framework for complete information. This dynamic programming method allows analysis of equilibria

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[2] Liu and Skrzypacz (2014) point out a number of such examples.

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when the time period is observed and for fixed discount factors, with a particularly simple interpretation when time is unobservable.

Under complete information when players have full memory, the dynamic programming methods of Abreu et al. (1990) (hereafter APS) characterize the equilibrium payoff set. The recursive structure of full memory games allows the equilibrium set to be calculated as the largest fixed point of a "generating operator," and is called the largest "self-generating set." DE extend this framework to bounded memory under complete information, showing that the appropriate self-generating notion is sets of payoff functions rather than payoffs. This paper extends these techniques to incomplete information, characterizing the set of weak Perfect Bayesian Equilibrium (wPBE) payoffs of repeated games where players learn about the world through private, possibly payoff-relevant states.[3] This setting includes reputation games where a single long-run player with full memory, who may be one of many types (e.g., commitment types), faces a sequence of short-run players who update their beliefs about the type from a limited number of recent periods, or a 1-to-5 star rating that evolves as the long-run player is rated by short-run players as on a review website. Alternatively, the framework allows multiple long-run players who each have private types and signals about the others' states, so long as they do not have full memory, forgetting what they observed in the past and only updating beliefs based on the current memory; an example might be competing firms who have limited institutional memory due to employee turnover.

The main challenge incomplete information presents is that in addition to playing best responses, players must also form beliefs consistent with Bayes' rule. Unlike full memory, tracking the posterior beliefs does not yield a recursive structure since these beliefs are forgotten.[4] Instead, the framework is generalized to a broader class of games where Nature chooses the initial states according to some probability distribution. Solving the whole set of such games together (i.e. for all probability distributions) yields a recursive structure that allows an analogous generating operator, as well as an algorithm for computing the largest fixed point by repeatedly applying the generating operator.

This framework is useful for games where time is observed. Previous papers studying bounded memory reputation games often hide the calendar date to restrict attention to stationary equilibria.[5] This is because although the strategies are time-independent, beliefs — and therefore best responses — may not be. When time is unobservable, equilibria have a simple interpretation as "self-generating *points*" rather than "self-generating sets." Although this makes the analysis much simpler, there are many real-world applications where even short-run players know the time: creditors know the age of borrowers even when credit histories are bounded, auto insurers know the age of drivers, and buyers can observe the age of a seller's account on eBay. The framework enables us to explore the impact of assuming unobservable-time.

Section 4 gives a reputation example that illustrates how the methods can be used in practice. The example is a product choice game between a long-run firm, where the firm may be an "honest" type committed to good treatment of consumers, playing against a sequence of short-run consumers who imperfectly monitor the firm's action through 1-period memory of a noisy signal. Much of the reputation literature following Fudenberg and Levine (1989, 1992) is concerned with the bounds on the worst long-run player equilibrium payoffs, particularly as the long-run player becomes very patient. The recursive tools are used to find the (actual) worst equilibrium payoffs under both observable and unobservable time, finding that non-stationary behavior allows worse equilibrium payoffs for the firm.

### *1.1. Related literature*

In addition to those mentioned above, this work relates to a variety of other papers on repeated games and reputation. A large literature has extended APS to many full memory models, including incomplete information settings like Cole and Kocherlakota (2001), Sleet and Yeltekin (2007), and Peski (2008) which also recursively ensure belief consistency. As mentioned above, a key difference is that full memory approaches use the fact that beliefs can be updated using just the previous period's posterior, which does not carry over to bounded memory.[6]

It also relates to a large literature on how restrictions on learning can affect commitment possibilities. Jehiel (2005) and Jehiel and Samuelson (2012) study "analogical reasoning" where players learn based on the average frequency of observations. Monte (2013) and Wilson (2014) study bounded memory as a finite set of memory states used by agents to construct strategies. Phelan (2006); Wiseman (2008); and Ekmekci et al. (2012) study reputation with impermanent player types. Ekmekci (2011) shows restricting learning through a finite rating system avoids the "temporary reputation" result of Cripps et al. (2004).

For complete information, other work on bounded memory includes the folk theorems of Barlo et al. (2009, 2016) and Mailath and Olszewski (2011). Cole and Kocherlakota (2005) study the equilibrium set of a complete information prisoner's dilemma with long finite memory. Imposing stationary behavior has been found to cause loss of equilibrium payoffs in a number of complete information bounded memory settings by Livshits (2002); DE; and Bhaskar et al. (2012).

[3] Public information can be encoded in these states as a special case.

[4] Note that bounded memory with complete information (the DE setting) is effectively a restriction on strategies, while bounded memory with incomplete information is a restriction on learning as well; i.e., for complete information, all bounded memory equilibria are also full memory equilibria, but not so for incomplete information.

[5] An exception is Ekmekci (2011) where players do know the date.

[6] Other properties like the martingale property also do not carry over to bounded memory. Indeed, the beliefs in non-stationary equilibrium found for the product choice game in Section 4 never converge.