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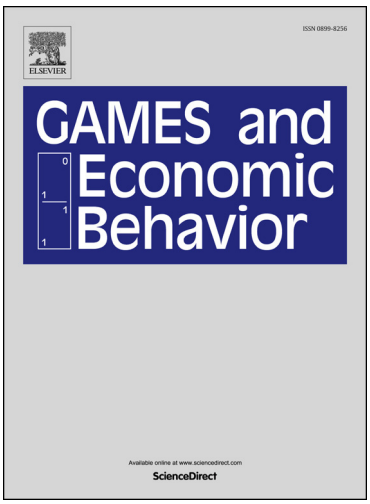

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GAMES OF THREATS

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Abstract. A game of threats on a finite set of players, N, is a function d that assigns a real number to any coalition, $S \subseteq N$, such that $d(S) = -d(N \setminus S)$. A game of threats is not necessarily a coalitional game as it may fail to satisfy the condition $d(\emptyset) = 0$. We show that analogs of the classic Shapley axioms for coalitional games determine a unique value for games of threats. This value assigns to each player an average of $d(S)$ across all the coalitions that include the player. Games of threats arise naturally in value theory for strategic games, and may have applications in other branches of game theory.

1. Introduction

The Shapley value is the most widely studied solution concept of cooperative game theory. It is defined on coalitional games, which are the standard objects of the theory.

A *coalitional game* on a finite set of players, N, is a function v that assigns a real number to any subset ("coalition"), $S \subseteq N$, such that $v(\emptyset) = 0$. The amount $v(S)$ may be interpreted as the *worth* of S, i.e., what the players belonging to S can jointly get by coordinating their efforts.

A *value* is a function that assigns to each coalitional game a vector of payoffs, one for each player, that reflects the a priori evaluation of each player's position in the game. In his celebrated paper [4] Shapley proposed four desirable properties ("axioms") and proved the remarkable result that there exists a unique function satisfying these properties. This function, the Shapley value, can be described as follows. The value of player i is an average of the marginal contributions, $v(S \cup i) - v(S)$, of player i, where the average is taken over all the orderings of N, with S denoting the subset of players that precede i in the ordering.

A *game of threats* is a function d that assigns a real number to any coalition, $S \subseteq N$, such that $d(S) = -d(N \setminus S)$. The amount $d(S)$ may be interpreted as the threat power of the coalition S, i.e., the maximal difference between the total amounts that the players belonging to S and the players belonging to $N \setminus S$ receive, when the players in S coordinate their efforts to maximize this difference and the players in $N \setminus S$ coordinate their efforts to minimize it. Games of threats arise naturally in value theory for strategic games [2].

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