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A note on the Shapley value for airport cost pooling game

Dongshuang Hou [a,∗,1], Hao Sun [a], Panfei Sun [a], Theo Driessen [b]

[a] Department of Applied Mathematics, Northwestern Polytechnical University, China
[b] Faculty of Electrical Engineering, Mathematics and Computer Science, Department of Applied Mathematics, University of Twente, The Netherlands

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ABSTRACT

The main goal of this paper is two-fold. First, we introduce the so-called airport cost pooling game, which is a generalization of the well-known class of airport game (Littlechild and Thompson, 1977). We determine the Shapley value of this class of game through a decomposition method for this game into unanimity cost games, exploiting the linearity of the Shapley value. Second, we characterize the Shapley value for airport cost pooling game by applying the so-called collective balanced contributions property, meaning that for any two airplanes from two different airlines, the withdrawal of one airline leads to the same loss to the airplane in the other.

1. Introduction

Game theory has been proved to be a helpful tool when analyzing the profit or the cost allocation problems. A mass of literature concentrating on this topic can be found. Examples include Littlechild and Owen (1973), who proposed the allocation of airport landing charges for different kinds of airplanes by determining the well-known airport game allocation, i.e., the Shapley value. Airport game deals with the problem of the determination of airplane landing costs. Several other papers have also conducted on this class of games, i.e., (Littlechild, 1974; Littlechild and Owen, 1976; Littlechild and Thompson, 1977; Dubey, 1977; Tijs and Driessen, 1986; Hou and Driessen, 2013). In these papers, all the airplanes are treated as independent units and the cost allocations are conducted by applying game theory. However, although the given model of airport game in the literature is very valuable, it is not taken into account the fact that airplanes are organized in airlines and are part of airlines. By observing this, Brage et al. (1997) studied this situation by proposing the airport game with prior unions, determined the Owen value (Owen, 1977) and explained the economic meaning of this value as an allocation of the airport construction cost.

In this paper we concentrate on the allocation of the airport construction cost by introducing the so-called airport cost pooling game. Note that, for any airline, each of its members is very powerful, i.e., if not all the members of this airline are present, then the agreement of construction of the airport from this airline can not be reached, yielding not any construction cost. In other words, the airplanes are organized in airlines, and incomplete airline has not any cost and no contribution to the cost of the other airlines. For this cost pooling situation, people may wonder how to distribute the total cost. To do that,

∗ Corresponding author.
E-mail address: dshhou@126.com (D. Hou).

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we conduct on the Shapley value, which is the average of the marginal contributions. The Shapley value, as one of the most well-known solution concepts, was introduced and characterized in Shapley (1953). As to the airport cost pooling game, of which the payoffs of the coalitions are measured by a maximum optimization problem, it is difficult to obtain the marginal contributions, which are involved in the Shapley value. Consequently, the traditional approaches existing in the literature to determine the Shapley value are not valid for this class of games. Technically, we determine the Shapley value through a decomposition approach for this game into unanimity cost games, which play a significant role to re-characterize the cost game. The strength of the Shapley value for airport cost pooling game determined in this paper is twofold. On one hand, the Shapley value is given by the gap of the construction costs of two distinct airlines, which directly points out the amount of the construction cost that the airplane should bear. On the other, our result makes it feasible to compute the amounts that the airplanes lose when the other airplanes or airlines withdraw from the game. Furthermore, the properties of this value can be conducted on based on our outcome, i.e., collective balanced contributions property.

To illustrate the fairness of the Shapley value, substantive axiomatizations can be found in the literature, for example, Shapley's original axiomatization by efficiency, null player property, additivity and symmetry (Shapley, 1953), Young's axiomatization by strong monotonicity (Young, 1985), and Chun's coalitional strategic equivalence (Chun, 1991). Also, Brage et al. characterized the Owen value by balanced contributions property and the quotient game property (Brage et al., 1997). However, in the context of cost pooling game, Shapley's initial axiomatization properties are out of work to characterize the uniqueness of the Shapley value since the airplanes from different airlines are not symmetric in the proposed unanimity games, resulting in the payoffs can not be determined. That is, the failure reason lies in the special structure of cost pooling situations. To characterize the values with special structure, in 2010, Calvo and Guttierrez generalized and proposed the coalitional balance contributions property for the coalitional value for the games with a coalition structure (Calvo and Gutierrez, 2010). In 2013, Kamijo came up with the so-called collective balanced contributions property, which can be used to characterize the collective value. Unlike the coalitional balance contributions property, the latter property judges at the individual level instead of considering the group's level with respect to the group's contributions (Kamijo, 2013). Thanks to the collective balanced contributions property, we axiomatize the Shapley value for airport cost pooling situations, in which this property is given new meanings. As to the cost pooling game, considering the fact that airplanes are organized in airlines, the collective balanced contributions property requires that, for every pair of airlines, the withdrawal of one airline will lead to the same loss to the airplanes in the other. Moreover, we show that in airport cost pooling situation, the collective balanced contributions property do characterize the Shapley value together with symmetry and efficiency.

The paper is organized as follows. In Section 2, we model the airport cost allocation situation as the airport cost pooling game. In Section 3, the Shapley value allocation is determined through a decomposition method for this game into unanimity cost games, Section 4 deals with the characterization of the Shapley solution for airport cost pooling situations. And the paper concludes with a brief summary and discussion of further research.

2. Airport game

Littlechild et al. have studied the situation of allocating airport construction charges by applying game theory. Generally, these costs have the identical characteristics that there are two kinds of costs.

(i) Variable costs from the taking-offs or landings by airplanes.

(ii) A fixed cost (e.g., terminal construction and runway construction).

In a general way, the variable costs are easy to be allocated because they are generated by individual airplanes. Therefore, the cost allocation problem degenerates into the problem of allocating the fixed cost. Lots of literature deal with the allocation of the fixed costs as follows: firstly define a cooperative cost game (N, C) of which the airplanes are treated as players and the cost of the coalition S is defined as the fixed cost incurring if a runway has to be build to offer service of landings and take-offs of all the airplanes in coalitions.

Although the existing research approaches of the cost allocation in airports in the literature are charming and helpful, one aspect of the situation is ignored, namely the airplanes at the airports are not independent, since the airplanes have agreements with airlines. Normally, the airplanes are grouped according to the airlines they are in, which renders that the lack of any members will lead to the failure of the agreement of construction of the airport. That is, if any members of the airline are not present, then the agreement of construction of the airport from this airline can not be reached, yielding not any construction cost. Based on this, we broaden and model the airport game by proposing the so-called airport cost polling game. As to the costs, similar to the cost structure mentioned in the literature, we require that the cost of constructing a runway depends on the airline with the largest construction cost of the runway to be built.

Definition 2.1. The airport cost pooling game is a tetrad (N, C, M, R), where $M = \{1, 2, \dots, m\}$ is the set of airlines, $R = (R_1, R_2, \dots, R_m)$ is the partition of the airplane set N with the understanding that R_i is the set of the members (airplanes) of the airline i, i.e., $N := \bigcup_{j=1}^{m} R_j$, and its characteristic cost function $C : 2^N \to R$ satisfying $C(\emptyset) = 0$ and

$$C(S) = \begin{cases} \max\limits_{k:R_k \subseteq S} c_k, & \text{if } \exists R_k \subseteq S, k \in M; \\ 0, & \text{otherwise}. \end{cases} \tag{2.1}$$