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Light scattering of particles illuminated by a divergent beam

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ABSTRACT

Analysis on particle size distribution is very important to a wide variety of industrial processes. A new measurement system of laser diffraction analyzer, in which a divergent beam is used as the incident light, is presented in this paper. Analytical expressions are obtained to describe the spatial distribution of the scattered light in the detection plane under or without the paraxial approximation. The results show that the new measurement system behaves in a similar way to those of the laser diffraction analyzers with reverse Fourier optics (RFO) and is especially suitable for measurements on small particles.

1. Introduction

Particle analysis is necessary to a wide variety of industrial processes, including grinding, crystallization, emulsification, and polymerization [1]. Information of particle size and particle size distribution (PSD) is valuable in the production of particles of specific sizes to control process efficiency and product quality. Instruments have been developed and constructed for this purpose. With the rapid development and availability of laser techniques, optoelectronics, and microcomputer science, light-scattering-based laser particle analyzers have found wide use since the late 1970s. This kind of instrumentation is characterized by accurate and fast measurement, a broad measurable size range, good reproducibility, and easy use.

Forward light scattering is one of the most widely used techniques to perform particle sizing on samples with particles close to or larger than the wavelength of light. The technology has been widely known as laser diffraction. However, the term laser diffraction no longer reflects the current state of the art. More general approaches based on the Mie theory and the measurement of scattering intensity over a wide scattering angular range is employed and hence the size range has been extended into the submicron region [2].

A typical schematic of the optical configurations for laser diffraction analyzers is shown in Fig. 1, wherein the particles are illuminated by a collimated beam or a convergent beam. The former is known as Fourier optics (FO) and the latter is the reverse Fourier optics (RFO). In the Fourier optics, particles pass through an expanded and collimated laser beam at some distances in front of a lens whose focal plane is positioned on a log-spaced ring-shaped detector. The lens focuses the incident beam so it will not interfere with the scattered light and it also transforms the angularly scattered light into a function of location in the detection plane. Usually, there is a small hole at the center of the ring-shaped detector which allows the incident light to pass through. The most important feature of the Fourier optics is that the scattered light at a specific angle is refracted by the Fourier lens onto a particular spot of the detector, regardless of the particle's location in the collimated beam. In the reverse Fourier optics, the relative positions of the particles and the Fourier lens are exchanged. The particles are illuminated by a convergent beam. Scattered light from particles is no longer collected by another lens; instead it is directly received by the ring-shaped detector. In the Fourier optics the effective focal length is that of the Fourier lens, while in the reverse Fourier optics the effective focal length is the distance between the particles and the center of the detector. So, a sample cell is used in reverse Fourier optics to limit the locations of particles, which defines the scattering volume.

The spatial distribution of the scattered light depends on the ratio of the particle size to the wavelength of the incident beam, the relative refractive index of the particles to the surrounding medium, and the effective focal length of the optical system. According to the diffraction approximation, the first maximum of the spatial distribution of the scattered light in the focal plane is located at $\pi ds/\lambda f_{eff}=1.357$, where d is the particle diameter, λ is the wavelength, f_{eff} is the effective focal length and s is the radius in the focal plane [3,4]. Thus, the range of particle sizes of the laser diffraction analyzers is determined by the effective focal length and the radii of the rings located at the inner and outer sides of the detector. Increasing the radius of the detector and reducing the effective focal length will extend the lower size limit

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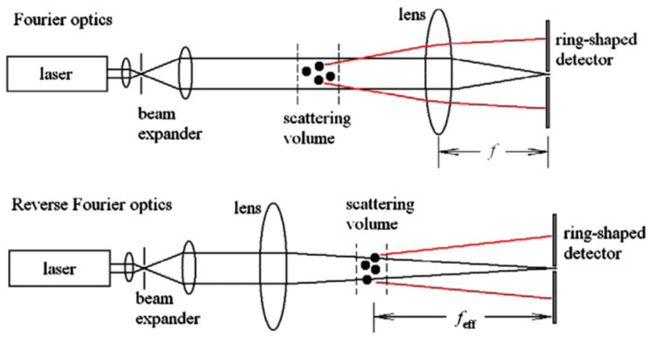

Fig. 1. Typical schematic of the optical configurations for laser diffraction analyzers.

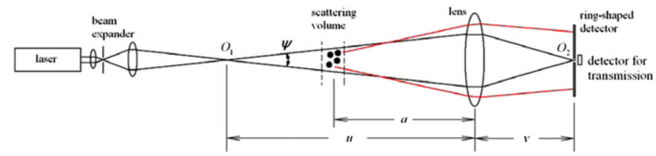

Fig. 2. Schematic of the optical configuration for particle analysis based on Fresnel diffraction.

of the laser diffraction analyzers. In order to measure small particles with the Fourier optics, the focal length of the Fourier lens has to be small. Nevertheless, the focal length of a lens cannot be reduced without limits and therefore the lower size limit of the laser diffraction analyzer using Fourier optics is limited. In the reverse Fourier optics, the effective focal length is the distance between the center of the scattering volume and that of the detector. So the lower size limit can be extended by moving the scattering volume towards the detector without changing the lens' focal length. The reverse Fourier optics allows measurements of much smaller particle sizes than the Fourier optics. However, while the scattering volume is very close to the detector, the size of the scattering volume should be very small [2] and disturbance from the multi-reflections of the scattered light between the sample cell and the detector on the signals will be enhanced, which finally causes measurement errors on the particle size distributions.

In this paper, we present a new measurement system of laser diffraction analyzer, in which a divergent beam is used as the incident light. In Section 2, the optical configuration and the measurement theory are introduced. The analytical expression of the spatial distribution of scattered light is first deduced by using the diffraction theory and/or the Mie theory under the paraxial approximation, and then a more accurate model is established. Numerical results and experimental evidences are given in Sections 3 and 4 together with some discussions on the characteristics of the measurement system.

2. Measurement theory

2.1. Outline of the optical configuration

A schematic of the optical configuration is shown in Fig. 2. A divergent beam (whose divergence is ψ) is used as the incident light and a convex lens is employed to focus the incident beam. The ring-shaped detector is coplanar to the focal point of the incident beam behind the lens. The ring-shaped detector is aligned such that the incident light may pass through the hole at the center of the detector. Behind the hole is attached a photodiode for the measurement of transmission. The scattering volume is positioned in front of the lens so that particles are illuminated by the divergent beam and the scattered light is transformed by the lens onto the ring-shaped detector.

2.2. Measurement principle based on diffraction theory

At first we consider the light pattern on the detection plane which is diffracted by a single particle. Under the paraxial approximation, the diffracted field in the detection plane is given as [5]

$$U_i(x,y)=\frac{C}{\lambda f_{eff}}\exp\left[-j\frac{\pi\mu}{\lambda}(x^2+y^2)\right]\iint U_{i,0}(\xi,\eta)\times\exp\left[-j\frac{2\pi}{\lambda f_{eff}}(x\xi+y\eta)d\xi\, d\eta\right] \tag{1}$$

where (x,y) is the coordinates in the detection plane, the subscript i denotes the number of the particle in the scattering volume, C is a constant, λ is the wavelength of the incident light, and the parameters μ and f_{eff} are defined as

$$\mu=\frac{(u+v)a-uv}{v^2(u-a)} \tag{2}$$

$$f_{eff}=\frac{u-a}{u-f}f \tag{3}$$

wherein u and v are denoted in Fig. 2 and they satisfy the relationship $u^{-1}+v^{-1}=f^{-1}$.

For a spherical particle whose diameter is d and whose center is located at $(\xi_{i,0},\eta_{i,0})$, we have

$$U_{i,0}(\xi,\eta)=\begin{cases}1 & \text{for}(\xi-\xi_{i,0})^2+(\eta-\eta_{i,0})^2\le(d/2)^2\\ 0 & \text{else}\end{cases} \tag{4}$$

So the solution of Eq. (1) may be obtained as

$$U_i(x,y)=\frac{C}{\lambda f_{eff}}\exp\left[-j\frac{\pi\mu}{\lambda}(x^2+y^2)-j\frac{2\pi}{\lambda f_{eff}}(x\xi_{i,0}+y\eta_{i,0})\right]\frac{2J_1(X)}{X} \tag{5}$$

where

$$X=\alpha\frac{\sqrt{x^2+y^2}}{f_{eff}} \tag{6}$$

$$\alpha=\frac{\pi d}{\lambda} \tag{7}$$

We assume that all particles located in the scattering volume are same in the size and are illuminated by the incident beam. The total field is thus given as

$$U(x,y)=\sum_{i=1}^{N}U_i(x,y)=\frac{C}{\lambda f_{eff}}\frac{2J_1(X)}{X}\sum_{i=1}^{N}\exp\left[-j\frac{\pi\mu}{\lambda}(x^2+y^2)-j\frac{2\pi}{\lambda f_{eff}}(x\xi_{i,0}+y\eta_{i,0})\right] \tag{8}$$

and the distribution of diffracted light intensity is

$$I(x,y)=\frac{C^2}{\lambda^2 f_{eff}^2}\left[\frac{2J_1(X)}{X}\right]^2\left|\sum_{i=1}^{N}\exp\left[-j\frac{\pi\mu}{\lambda}(x^2+y^2)-j\frac{2\pi}{\lambda f_{eff}}(x\xi_{i,0}+y\eta_{i,0})\right]\right|^2 \tag{9}$$

where N is the number of particles in the scattering volume.

If the particle concentration is so low that the interaction between particles can be neglected, the particles locate in the scattering volume randomly. Therefore, the interference between the fields diffracted by the single particles fades away and hence