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Effectivity and power

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ABSTRACT

We axiomatically develop a class of power indices for effectivity functions, both for the case where the set of alternatives is finite and where it is infinite. Such power indices make it possible to take the issues under consideration into account, in contrast to power indices defined just for simple games. As an example, we consider the US legislative system. We also show that our approach can be used to develop power indices for spatial political games.

1. Introduction

A power index is a tool to measure and compare the power of players in political and economic situations. In a democratic Parliament the number of seats of a political party is a poor indication of its power. Rather, a measure of power should be based on the likelihood that such a party is decisive. One way to assess this is to model Parliament as a simple game, by establishing which coalitions of political parties are winning and which are losing – based on the required majorities to pass bills or amendments. To such a simple game one may then apply a power index, for instance the Banzhaf–Coleman index and the Shapley–Shubik index (see for instance Bertini et al., 2013, for a recent overview).

Similarly, economic situations as for instance financial and corporate governance structures may be modelled by simple games, and power indices may be applied in order to obtain some indication of the real influence of firms or investment companies in such situations (Crama and Leruth, 2007, 2013; Karos and Peters, 2015).

These indices are ex ante power measures: their main application lies in the design of voting mechanisms that have a desired distribution of power among its members (see for instance Felsenthal and Machover, 2005; Laruelle and Valenciano, 2008, for details). This approach to power measurement clearly has its limitations: In political situations, neither the position of a political party nor the issues at stake are taken into consideration. For this reason, in political science spatial models are considered, which enable to represent both a party's position and the issues involved.

In the present paper we generalize the simple game approach and also the spatial game approach by considering the very broad model of an effectivity function (Moulin and Peleg, 1982) and developing a theory of power indices for such functions. Given a number of players and a set of alternatives, an effectivity function assigns to each coalition (subset of players) a collection of sets of alternatives. If a coalition S is effective for a set B of alternatives, then this is generally interpreted as the coalition S being able to guarantee that the final alternative is in B. Alternatives could be social states, and a coalition may be constitutionally entitled to the final social state being in a certain set of social states. Or alternatives could be laws, and a coalition of parties may have the power (e.g., because it has a certain majority) to pass certain laws. In financial or

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governance structures, coalitions of shareholders may enforce certain decisions within companies if they own sufficiently many shares. More formally, effectivity functions may be derived from social choice functions or correspondences, or from game forms.

The following is a specific political example, which will recur at several places in this paper.

Example 1.1 *(The US legislative process).* In US legislation any bill must be passed by the two chambers of Congress, the Senate (100 members) and the House (of Representatives, 435 members), and signed by the President before it comes to law. If the President vetoes the bill, Congress can override this decision by a two third majority. In case of an Executive Order by the President, no approval by Congress is needed.

The President can agree on international treaties, but these must be approved by a two third majority of the Senate. In case of Congressional-executive Agreements (agreements that affect domestic policies, such as free trade agreements) a simple majority approval both in the Senate and in the House are necessary; and Executive Agreements need no approval by Congress at all.

Before and after the 2016 election the Republicans have (had) a majority in both chambers of Congress, but they do not have a two third majority. If we consider the Republicans (R) and the Democrats (D) as one player each, we see that until the new President is sworn in, the singleton set $\{D\}$ is effective for any outcome that can be reached via Executive Orders or Agreements, while $\{R\}$ is not effective for anything. After that $\{R\}$ will be effective for outcomes that can be reached via normal bills, Congressional-executive agreements, and Executive Orders and Agreements.

Alternatively, we can analyze the situation from the individuals' points view. In this case the singleton $\{p\}$ which contains only the president, is effective for any outcome that can be reached via Executive Orders or Agreements; a coalition S is effective for an outcome $\{a\}$ that can be reached via a domestic law or a Congressional-executive Agreement if S contains p and more than half of the members of the Senate and of the House. In the first case it is also sufficient if S contains more than two thirds of the members of the Senate and of the House. In particular, a coalition T is effective for the set $A \setminus \{a\}$ – in other words, T can veto a – if $p \in T$ and, in case of a domestic law, T contains more than one third of the members of either the Senate or the House. If a is an international treaty then S is effective for $\{a\}$ if $p \in S$ and S contains two thirds of the members of the Senate. □

Effectivity functions and the concept of a power index for effectivity functions are introduced in Section 2. Our basic axioms for power indices are introduced in Section 3: the Transfer Property, Anonymity, and Monotonicity. The Transfer Property is a generalized additivity condition which enables us to unravel an effectivity function in terms of elementary effectivity functions which are much easier to analyze. Anonymity and Monotonicity are natural conditions in this context. In Section 4 we consider the case where the set of alternatives is finite and characterize all power indices satisfying the Transfer Property, Anonymity, and Monotonicity. Such a power index works as follows. For each set of alternatives, consider the simple game in which exactly those coalitions that are effective for that set of alternatives are winning. Also, to each set of alternatives attach a non-negative number such that these numbers sum up to one. Then the power index is obtained by taking the sum of the Shapley values of these simple games weighted by the chosen numbers. This leaves much freedom, namely in choosing these weights. In the remainder of Section 4 we (1) show that all these power indices reduce to the Shapley value for an effectivity function associated with a simple game (winning coalitions are effective for every set of alternatives, losing coalitions only for the whole set); (2) establish connections between power indices for the duals of simple games and polar effectivity functions (which describe what coalitions can obtain in reaction to outside players); (3) show that a power index is neutral if and only if the weights only depend on the cardinalities of the sets of alternatives; (4) show that requiring a stronger version of Monotonicity results in non-singleton nontrivial sets of alternatives having weight zero; and (5) consider the role of null-players. In Section 5 we extend our main characterization to effectivity functions for an infinite set of alternatives, under Strong Monotonicity instead of Monotonicity and, additionally a Null Player axiom and a continuity condition. Basically, we obtain a similar characterization, now with the weights of the finite case replaced by a probability measure over the set of alternatives. In a separate subsection, we show how this approach can lead to developing power indices for spatial political models, similar in spirit to the Owen–Shapley spatial power index.

In an Appendix we show that the axioms in our main characterization are logically independent.

2. Preliminaries

We start with some notations. For a set D we denote by $P(D)$ the set of all subsets of D, and by $P_0(D)$ the set of all nonempty subsets of D. By $|D|$ we denote the number of elements of D.

Throughout, $N = \{1, \dots, n\}$ ($n \in \mathbb{N}$) is the set of *players*. Subsets of N are also called *coalitions*. We denote by A the set of *alternatives*. We fix a set $\mathcal{T} \subseteq P_0(A)$. In the first part of the paper, A is finite and $\mathcal{T} = P_0(A)$; later, A can be infinite, endowed with a topology, and then $\mathcal{T}$ will be the collection of nonempty closed subsets of A.

Definition 2.1. An *effectivity function* (for $\mathcal{T}$) is a map $E : P(N) \to P(\mathcal{T})$ such that (i) $P(\emptyset) = \emptyset$, (ii) $A \in E(S)$ for every $S \in P_0(N)$, and (iii) $E(N) = \mathcal{T}$. An effectivity function E is *monotonic* if $B \in E(S)$ implies $B' \in E(T)$ for all $B, B' \in \mathcal{T}$ and $S, T \in P_0(N)$ such that $B \subseteq B'$ and $S \subseteq T$. An effectivity function E is *superadditive* if $B \cap B' \in E(S \cup T)$ for all $B, B' \in \mathcal{T}$ and $S, T \in P_0(N)$ such that $B \in E(S)$, $B' \in E(T)$, and $S \cap T = \emptyset$. By $\mathcal{E}$ we denote the set of all monotonic and superadditive effectivity functions.

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