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Graphical exchange mechanisms ☆

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ABSTRACT

Consider an exchange mechanism which accepts "diversified" offers of various commodities and then redistributes them. Under some natural conditions of "fairness" and "convenience", such a mechanism admits unique prices, which equalize the value of offers and returns for every individual. Next define the complexity of a mechanism via certain integers τ_{ij}, π_{ij} and k_i that represent the time required to exchange i for j, the difficulty in determining the exchange ratio, and the dimension of the offers. There are *finitely* many minimally complex mechanisms, in each of which all trade occurs through markets for commodity pairs. Finally consider minimal mechanisms with smallest worst-case complexities $\tau = \max \tau_{ij}$ and $\pi = \max \pi_{ij}$. For $m > 3$ commodities, there are precisely three such mechanisms, one of which has a distinguished commodity – the money – as the sole medium of exchange. As $m \to \infty$ the money mechanism is the only one with bounded (π, τ).

1. Introduction

The purpose of this paper is to show how simple criteria of fairness, convenience and complexity can lead[1] to the successive emergence of prices, markets, and money, in a Cournotian setting for commodity exchange. In the process, we arrive at a rationale for money which is purely "mechanistic" in spirit, and complements the existing utilitarian and behavioral literature on the subject (see 1.1).

We consider abstract exchange mechanisms,[2] which accept "diversified" offers of various commodities and redistribute everything, and further satisfy five conditions, embodying fairness and convenience that we term *Anonymity, Aggregation,*

☆ In honor of Lloyd Shapley. (The authors thank David Levine and two anonymous referees for suggestions that have improved the presentation.)
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[1] I.e., by way of an abstract model that captures the transition from a barter to a monetary economy, and not in terms of specific historical or geographical developments.

[2] For us a Cournotian "mechanism" is a formal device that enables everyman to trade, with the simple expedient of offering commodities and without having to account for his precise motivation or even bothering to pretend that he has one. (See section 1.1.3.1 of Mertens, 2003 for a spirited defense of the use of "mechanism" in this plain English meaning of the word.) To forestall confusion, we emphasize that our usage is different from that of the recent mechanism-design literature, where the word has acquired a specialized connotation.

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Invariance, Non-dissipation and *Flexibility*. Although there are infinitely many such mechanisms, our first result is that each admits unique *prices*,[3] which lead to *value conservation*, i.e. equalize the value of individual offers and returns.

We next define some natural notions of "complexity" for a mechanism, and, in keeping with the idea of convenience, study mechanisms with minimal complexity. This leads to a *finite* class $\mathfrak{M}_* \subset \mathfrak{M}_g$, where $\mathfrak{M}_g$ denotes certain *graphical mechanisms* that are in one-to-one correspondence with directed, connected graphs on the set of commodities. The directed edge ij may be interpreted in $\mathfrak{M}_*$ as a *market* that provides traders the opportunity to offer i in exchange for j. Prices not only conserve values in $\mathfrak{M}_*$ (in fact, in $\mathfrak{M}_g$) but *mediate trade* in the sense that the return to a trader depends only on his own offer and the prices. In short, prices "decouple" the interaction between traders.

The emergence of prices and markets paves the way for the culmination of the analysis, namely the emergence of *money*. To this end we introduce additional refined criteria of complexity on $\mathfrak{M}_*$ and study the corresponding minimal mechanisms, which we term *strongly* minimal. It turns out that there are only *three* strongly minimal mechanisms, up to a relabeling of commodities. In one of these, a single commodity emerges endogenously as *money* and mediates trade among decentralized markets for the other commodities. Moreover, with a moderate increase in the number of commodities, the money mechanism quickly supersedes the other two in a very precise sense.

There is a companion (Dubey et al., 2015) to this paper, but the two are meant to be readable independently. This has necessitated some overlap but it is minimal. To be precise, the conditions on the mechanism (with the exception of Flexibility) appear in Dubey et al. (2015), so does Proposition 5, and the rest is disjoint.

Finally let us mention that both papers are in fond memory of Lloyd Shapley, whose profound contributions have shaped many areas of Game Theory, including strategic market games.

1.1. *Related literature*

The emergence of money as a medium of exchange has been a matter of considerable discussion in economics. One approach, following Jevons (1875), has focused on search-theoretic models that involve repeated and random bilateral meetings between agents (see, e.g., Bannerjee and Maskin, 1996; Iwai, 1996; Jones, 1976; Kiyotaki and Wright, 1989, 1993; Li and Wright, 1998; Ostroy, 1973; Trejos and Wright, 1995 and the references therein). Another line of inquiry is based on partial or general equilibrium models with various kinds of frictions in trade, such as transactions costs or limited trading opportunities (see, e.g., Foley, 1970; Hahn, 1971; Heller, 1974; Heller and Starr, 1976; Howitt and Clower, 2000; Norman, 1987; Ostroy and Starr, 1974, 1990; Starr and Stinchcombe, 1999; Starr, 2012; Starret, 1973; Wallace, 1980). These models turn on notions of rational expectations and utility-maximizing behavior of the agents in equilibrium. In contrast, as was said, our analysis is based purely on the mechanism of trade and is independent of the characteristics of the agents, such as their endowments or utilities.

It is worth emphasizing that our analysis is quite agnostic regarding the choice of any particular money,[4] being only at pains to point out the urgency of appointing *some* money. For a discussion of different criteria entailed in the choice of a suitable "commodity money" such as its portability, reliability, divisibility and durability; or, alternatively, the backing of the state requisite to sustain "fiat money", see, e.g., Smith (1776), Jevons (1875), Knapp (1905), Kaulla (1920), Lerner (1947), Menger (1892), Schumpeter (1954); and, for a recent survey on both kinds of money, see Shubik (1999) and Starr (2012). It would be interesting to incorporate some of these criteria, as well as the utilitarian considerations for money, into our mechanistic framework.

This paper is intimately related to Dubey et al. (2015). Let us briefly recount the model there. We make the *hypothesis* in Dubey et al. (2015) that any offer of a commodity i specifies some other commodity j which is being sought in exchange for i. Thus, drawing a directed edge ij for every such trading opportunity permitted in the mechanism, we obtain a (directed, connected[5]) graph G whose vertex set corresponds to the commodities; and the offer of any trader in commodity i is tantamount to placing weights on the outgoing edges at i. The role of a mechanism is simply to redistribute all the commodities it has been offered. There are clearly infinitely many mechanisms for any given G, but it is shown in Dubey et al. (2015) that exactly one of them is categorically determined by *four* of our five conditions, namely *Anonymity, Aggregation, Invariance, Non-dissipation* (all except *Flexibility*). This is the graphical "G-mechanism" mentioned earlier; so that the class $\mathfrak{M}_g$ is precisely the one generated as G varies over all directed, connected graphs on the fixed vertex-set of commodities. Our refined complexity criteria apply equally to $\mathfrak{M}_g$ as to its subset $\mathfrak{M}_*$, and we show that $\mathfrak{M}_g$ has the *same* three strongly minimal mechanisms as $\mathfrak{M}_*$. This is the main conclusion of Dubey et al. (2015) and it constitutes a key step in the proof of the emergence of money in this paper.

Our purpose here is to put the analysis of Dubey et al. (2015) on a general footing. We start with a domain that includes $\mathfrak{M}_g$, but is considerably richer. To begin with, the offers of any commodity $i = 1, \ldots, m$ are no longer constrained to point to other commodities $j \neq i$ as in $\mathfrak{M}_g$, but they can be much more "prolific". In mathematical terms, they correspond to

[3] I.e., consistent exchange-rates across commodity pairs.

[4] Our model can equally accommodate fiat money or commodity money, depending on how preferences are introduced. Indeed, *all* we suppose is that the m items being traded are distinct from each other. In particular, offers and returns could just be quotes (think of e-commerce!), instead of actual shipment of goods.

[5] Connected, because we require that it should be possible to convert any commodity to another by iterated trading.