ARTICLE IN PRESS
YGAME:2759

Games and Economic Behavior ••• (••••) •••–•••

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Coalition preferences with individual prospects ☆

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ARTICLE INFO

Article history:
Received 8 August 2017
Available online xxxx

JEL classification:
D60
D70
D71
D81

Keywords:
Preference aggregation
Incomplete preferences
Extended Pareto rule

ABSTRACT

We consider a group of individuals, such that each coalition of them is endowed with a preference relation, which may be incomplete, over a given set of prospects, and such that the extended Pareto rule holds. We assume that each singleton coalition has complete vNM preferences. In this setup, Baucells and Shapley (2008) gave a sufficient condition for a coalition to have complete preferences, in terms of the completeness of preferences of certain pairs of individuals. The new property that we introduce of *individual prospects* requires each individual to have a pair of consequences between which only she is not indifferent. We show that with this property a weaker condition guarantees the completeness of preferences of a coalition: it suffices for a coalition to be a union of a connected family of coalitions with complete preferences.

1. Introduction

Consider a group of individuals each possessing a complete vNM preferences over a set of prospects. We further assume that each coalition of individuals is endowed with preferences that may be incomplete, i.e., some prospects may be incomparable. If we think of the preference relation of a coalition as an aggregation of the preferences of its members then the incompleteness of the coalition's preference is the result of the inability to socially compare certain alternatives. Our aim is to provide sufficient conditions for the completeness of the preferences of a coalition in terms of the complete preferences of some of its subcoalitions.

We assume that the coalition preference relations satisfy the Extended Pareto (EP) rule: if two disjoint coalitions agree on the preference relation between two prospects, then the union of these coalitions also has the same preference over the prospects. The EP rule was introduced in Shapley and Shubik (1974, p. 65), and explored by Dhillon (1998), Dhillon and Mertens (1999), Baucells and Sarin (2003) and Baucells and Shapley (2008). It extends and implies the Pareto requirement in Harsanyi (1955), which requires that when all individuals agree on the preference relation between two prospects, then the grand coalition also agree with this preference relation.

☆ This paper builds on Baucells and Shapley (2008), and we are indebted to Lloyd Shapley, who contributed to the start of the project. A prior working version of this paper had the title "Multiperson Utility: The Linearly Independent Case." Samet acknowledges financial support of the Israel Science Foundation, grant 520/16, and the Henry Crown Institute of Business Research in Israel.

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https://doi.org/10.1016/j.geb.2017.10.006
0899-8256/

Please cite this article in press as: Baucells, M., Samet, D. Coalition preferences with individual prospects. Games Econ. Behav. (2017), https://doi.org/10.1016/j.geb.2017.10.006

The following claim, which is a simplified version of the main result of Baucells and Shapley (2008), provides a sufficient condition for a coalition S to have complete preferences.[1]

($*$) Consider a connected graph the vertices of which are all the individuals in S. If each pair of individuals that form an edge in this graph has complete preferences, then S also has complete preferences.

Here, in addition to the EP rule, we assume the existence of Individual Prospects (IP). That is, for each individual there exists a pair of prospects, such that the individual prefers one to the other, while all other individuals are indifferent between them. If these prospects are lotteries of prizes this assumption holds when for each individual there is a prize that matters only to him. If prizes are monetary, then the assumption would require there to be more prizes than individuals, and have sufficient diversity among individuals' risk preferences. Assuming IP enables us to give the following sufficient condition for the completeness of the preferences of S.

($**$) Consider a connected graph, the vertices of which are subcoalitions of S, such that the union of these coalitions is S and each pair of coalitions that form an edge in the graph has a non-empty intersection. If each of these coalitions has complete preferences then S also has complete preferences.

The graphs in ($*$) and ($**$) are different. The nodes in the first are individuals, while in our condition, ($**$), they are coalitions. However, the restriction of ($**$) to coalitions of size two is equivalent to ($*$). Thus, our condition ($**$) is more general, and therefore weaker. We manage to reach the same conclusion as Baucells and Shapley (2008) with a weaker condition because of the assumption of IP.

Representation of preferences in terms of linear functions on the prospects plays an important role in our analysis and proofs. Representations of incomplete preferences were developed by Aumann (1962), Shapley and Baucells (1998), and Seidenfeld et al. (1995), and were revisited by Dubra et al. (2004) and Galaabaatar and Karni (2012). Incomplete preferences can be described by cones of utility vectors in the dual space of the space that contains the prospects. For complete preferences the cone is a one dimensional ray. The EP rule can also be described in terms of the cones defining the various coalitional preferences. The IP condition introduced here is equivalent to the requirement for the utility vectors of the individuals to be linearly independent. Linear independence of the utility vectors of each triplet of individuals was assumed in Baucells and Shapley (2008). However, the latter property is expressed in terms of the representation of the preferences and not in terms of the preferences themselves.

2. The main result

We consider a set of prospects $\mathcal{M}$, which is a full-dimensional, closed, convex subset of $\mathbb{R}^m$.[2] An *incomplete preference relation* on $\mathcal{M}$ is a binary relation $\succsim \subseteq \mathcal{M} \times \mathcal{M}$ that is reflexive ($\forall p$, $p \succsim p$), transitive ($\forall p, q, r$, if $p \succsim q$ and $q \succsim r$, then $p \succsim r$), continuous (the set $\{\alpha : p \succsim \alpha q + (1-\alpha) r\}$ is closed), and satisfies the axiom of independence ($\forall p, q, r$ and $\alpha \neq 0$, $p \succsim q$ iff $\alpha p + (1-\alpha) r \succsim \alpha q + (1-\alpha) r$). The relations $\sim$ and $\succ$ are defined as usual. The preference relation is *complete* when for all p and q, either $p \succsim q$ or $q \succsim p$. The *trivial preference* is the complete preference that satisfies $p \succsim q$ for all prospects p and q, and thus $p \sim q$ for all p and q.

Let $N = \{1, \ldots, n\}$ be a set of individuals. Non-empty subsets of N are called *coalitions*. With some abuse of notation we write i for $\{i\}$. We implicitly assume that all individuals agree on $\mathcal{M}$, which is the case when probabilities are objective.[3]

Definition 1. A *coalition preference* is an assignment of an incomplete preference relation to each coalition S, denoted by $\succsim_S$, such that for each individual i, $\succsim_i$ is complete.

We assume that the coalition preference satisfies the following two properties:

Extended Pareto (EP)

For all disjoint coalitions A and B, and for all $p, q \in \mathcal{M}$, if $p \succsim_A q$ and $p \succsim_B q$, then $p \succsim_{A \cup B} q$, and if $p \succ_A q$ and $p \succsim_B q$, then $p \succ_{A \cup B} q$.

[1] Their theorem is formulated for the grand coalition, but it trivially extends to any coalition S. Also, their condition appears to be stronger than ($*$), since it is required to hold for certain small graphs. It is equivalent to the condition presented here, as we show in the discussion that follows Example 1 below. Their theorem also requires a technical "triplet linear independence" assumption, which we discuss below.

[2] For example, $\mathcal{M} = \left\{p \in \mathbb{R}^m : \sum_{k=1}^{m} p_k \leq 1,\, p_k \geq 0\right\}$ could represent probability mixtures between $m+1$ outcomes. In this case and for $m = 2$, $\mathcal{M}$ is the Marshak triangle.

[3] Having agreement on probabilities puts aside the dilemma between maintaining the Pareto rule but having no group beliefs (Hylland and Zeckhauser, 1979; Mongin, 1995; Nau, 2006), or keeping group beliefs but violating the Pareto rule when individual beliefs differ (Gilboa et al., 2004).

Please cite this article in press as: Baucells, M., Samet, D. Coalition preferences with individual prospects. Games Econ. Behav. (2017), https://doi.org/10.1016/j.geb.2017.10.006