Games and Economic Behavior 106 (2017) 317–328

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Sequential versus static screening: An equivalence result

Daniel Krähmer [a,*], Roland Strausz [b,1]

[a] Universität Bonn, Institut für Mikroökonomik, Adenauer Allee 24-42, D-53113 Bonn, Germany
[b] Humboldt-Universität zu Berlin, Institute for Economic Theory 1, Spandauer Str. 1, D-10178 Berlin, Germany

ARTICLE INFO

Article history:
Received 13 March 2017
Available online 26 October 2017

JEL classification:
D82
H57

Keywords:
Sequential screening
Static screening
Stochastic mechanisms

ABSTRACT

We show that every sequential screening model is equivalent to a standard text book static screening model. We use this result and apply well-established techniques from static screening to obtain solutions for classes of sequential screening models for which standard sequential screening techniques are not applicable. Moreover, we identify the counterparts of well-understood features of the static screening model in the corresponding sequential screening model such as the single-crossing condition and conditions that imply the optimality of deterministic schedules.

1. Introduction

Recent years have witnessed an increased interest in dynamic adverse selection models in which agents receive novel private information over time. One of the most fundamental dynamic adverse selection models, the so-called sequential screening model, has been introduced by Courty and Li (2000). In this model, a seller offers a single good for sale, but in contrast to a static environment, the buyer initially has private information only about the distribution of his valuation, and he fully learns his valuation only after contracting has taken place. Due to its analytical tractability, the sequential screening model has become the workhorse model for analyzing various applied dynamic contracting problems such as ticket pricing, dynamic procurement, or the sale and disclosure of information.[2]

Dynamic adverse selection models are not only practically relevant, they also raise interesting conceptual questions about their relation to well-understood static adverse selection models. In this paper, we demonstrate that for the sequential screening model this relation is actually surprisingly tight. More specifically, we show that any sequential screening model can be equivalently represented as a canonical textbook static screening model (as, e.g., described in Fudenberg and Tirole, 1991) so that any contract which is feasible (resp. optimal) in one problem is also feasible (resp.

* Corresponding author.
E-mail addresses: kraehmer@hcm.uni-bonn.de (D. Krähmer), strauszr@wiwi.hu-berlin.de (R. Strausz).

1 We thank two anonymous referees and the editor for very helpful suggestions. We are also grateful to Holger Herbst, Johannes Hörner, Matthias Lang, and Urs Schweizer for useful comments. We also thank seminar participants at Paris, Dauphine ("Topics in Information Economics"), in Berlin ("Theoretischer Ausschuss"), Bonn, Bristol, Helsinki, and Mannheim. Roland Strausz gratefully acknowledges financial support by the DFG (German Science Foundation) under SFB/TR-190.

2 For applications of sequential screening models, see Dai et al. (2006), Esö and Szentes (2007a, 2007b), Hoffmann and Inderst (2011), Nocke et al. (2011), Krähmer and Strausz (2011, 2015a, 2015b), Inderst and Peitz (2012), Bergemann and Wambach (2015), Deb and Said (2015), Liu and Lu (2015), Li and Shi (2017). For a textbook treatment, see Krähmer and Strausz (2015c). For more general models of dynamic adverse selection, see Baron and Besanko (1984), Battaglini (2005), and Pavan et al. (2014).

https://doi.org/10.1016/j.geb.2017.10.015
0899-8256/

optimal) in the other. Reversely, we identify a class of static screening models that each correspond to an appropriate sequential screening model.

Sequential screening problems are best understood in so-called regular environments which require that the (sequential) virtual trade surplus satisfies a certain monotonicity condition. In contrast, little is known for non-regular environments. In the second step of our analysis, we show that our equivalence result can be used to obtain solutions for classes of non-regular sequential screening problems. More precisely, we identify conditions for which the sequential screening problem is not regular, but the corresponding static screening problem can be solved with well-known techniques from static screening.[3]

Moreover, our equivalence result clarifies the role of the ordering of the agent's private information in sequential screening models. The most often used ordering in sequential screening is that a buyer type is higher if he is more likely to obtain favorable subsequent information in the sense of first order stochastic dominance. As it turns out, the equivalent condition in the corresponding static screening model is precisely that the agent's utility function satisfies the single-crossing condition. Reversely, a sequential screening problem in which types are not ordered according to first order stochastic dominance corresponds to a static screening problem without single-crossing.

Key in establishing the connection between the sequential and the static model is to explicitly allow for the use of stochastic contracts in the static model. Intuitively, stochastic contracts enter the picture, because in the sequential model the terms of trade depend on the buyer's valuation that realizes ex post and are, from an ex ante perspective, therefore stochastic. Our insight is that the induced distribution of terms of trade can be replicated by a stochastic contract in the static model so that a party's expected utility in the sequential model, where the expectation is taken with respect to the buyer's future valuation, coincides with its expected utility in the static model, where the expectation is taken with respect to the uncertainty generated by the stochastic contract. By allowing for stochastic contracts, our equivalence result also sheds light on their optimality in sequential screening environments.

It is worth mentioning that our result is *not* implied by a general principle, such as, for example, Pontryagin's maximum principle which states that a dynamic optimization problem can be reduced to *some* static problem subject to *some* constraints. Rather, the insight of our paper is more specific and, therefore, more surprising: the sequential screening model can be represented as a very specific static model, namely exactly as the familiar standard principal agent adverse selection model.

The paper is organized as follows. The next section introduces the two models and derives our equivalence result. Section 3 applies the equivalence result to non-regular sequential screening problems, and Section 4 concludes.

2. Sequential versus static screening

2.1. The sequential screening problem

This subsection considers the sequential screening model of Courty and Li (2000). There is a buyer (the agent, he) and a seller (the principal, she), who has a single unit of a good for sale. The buyer's valuation of the good is $x \in [0,1]$, and the seller's opportunity costs are $c \geq 0$. The terms of trade specify the probability $q \in [0,1]$ with which the good is exchanged and an expected payment $t \in \mathbb{R}$ from the buyer to the seller. The parties are risk neutral and have quasi-linear utility functions. That is, the seller's profit equals payments minus her expected opportunity costs, $t - cq$, and the buyer's utility equals his expected valuation minus payments, $xq - t$. Each party's reservation utility is normalized to 0.

There are three periods. At the contracting stage in period 1, no party knows the buyer's true valuation, but the buyer privately knows that his valuation x is distributed according to the distribution function $G(x|\theta)$ on the support $[0,1]$ with density $g(x|\theta)$. While the buyer's *ex ante type* θ is his private information, it is commonly known that θ is drawn from the distribution $F(\theta)$ with support $[0,1]$ and density $f(\theta)$. In period 2, after the buyer has accepted the contract, the buyer privately observes his true valuation x. We refer to x as the buyer's *ex post type*. Finally, in period 3, the contract is implemented. We allow the seller's opportunity costs $c = c(\theta, x)$ to depend on the buyer's types.[4]

The seller's problem is to design a contract that maximizes her expected profits. By the revelation principle for sequential games (e.g., Myerson, 1986), the optimal contract can be found in the class of direct and incentive compatible contracts which, on the equilibrium path, induce the buyer to report his type truthfully. Formally, a *direct contract*

$$\gamma^d \equiv \{(q^d(\hat{\theta},\hat{x}), t^d(\hat{\theta},\hat{x})) | (\hat{\theta},\hat{x}) \in [0,1]^2\} \tag{1}$$

requires the buyer to report an ex ante type θ in period 1, and an ex post type x in period 2. A contract commits the seller to a *selling schedule* $q^d(\hat{\theta},\hat{x})$ and a *transfer schedule* $t^d(\hat{\theta},\hat{x})$.

If the buyer's true ex post type is x and his period 1 report was $\hat{\theta}$, then his utility from reporting $\hat{x}$ in period 2 is

$$\tilde{u}(\hat{x}|\hat{\theta}, x) \equiv x q^d(\hat{\theta},\hat{x}) - t^d(\hat{\theta},\hat{x}). \tag{2}$$

[3] While the solutions we obtain may involve bunching, we show how an approach developed in Nöldeke and Samuelson (2007) can be applied in our setting, which does not require optimal control techniques to identify optimal bunches.

[4] Our equivalence result in Propositions 1 and 2 below goes through for discrete (ex ante and/or ex post) type spaces as well. For tractability reasons, the application in Section 3, where we illustrate the usefulness of our equivalence result, is developed for continuous type spaces.