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Lloyd Shapley and chess with imperfect information ☆

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ABSTRACT

Anyone who has ever studied game theory knows the name Lloyd Shapley. Just recall Matching, Deferred-Acceptance Algorithm, Core, Market Games, Stochastic Games, Shapley value, and Shapley vector.[1] But Professor Shapley was also a great lover of chess with imperfect information. Upon our first encounter at Stony Brook in 1998, I was fortunate to investigate the chess problems he set before me. In this essay I analyze some of those problems, in commemoration of Lloyd Shapley's contributions to the study of chess and chess with imperfect information.

"Talent hits a target no one else can hit. Genius hits a target no one else can see."

Arthur Schopenhauer

1. Preliminary: chess with imperfect information

From the perspective of game theory, chess is not of great interest because it is ultimately a zero-sum game with perfect information: each player knows all previous moves, and may construct (so far only in theory) the complete game tree.[2,3] Therefore, using backward induction, it is mathematically feasible for any player to find a subgame perfect equilibrium.[4] Nevertheless, the vast number of possible decision nodes has thus far prevented even the most powerful computers from completing such an analysis.

An additional layer of complexity may be introduced to chess by limiting the information available to its players. Chess with imperfect information, or Kriegspiel, was introduced in Germany at the end of the 19th century,[5] and it is this more complex game which stimulated Professor Shapley's interest. In Kriegspiel each player can see their own pieces, but not

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[1] See also Serrano (2013, 2017) and http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/2012/shapley-facts.html.
[2] Rules of chess can be found at https://www.fide.com/component/handbook/?id=207&view=article.
[3] The "triviality" of chess was discussed in the literature. See, for example, Zermelo (1913), Ewerhart (2000), and Schwalbe and Walker (2001).
[4] Even though most people assume that chess is a finite game, this is formally not the case. See Ewerhart (2002).
[5] Nasar (1998) also noted that Kriegspiel was popular at Princeton when John Nash and Lloyd Shapley were Ph.D. students there.

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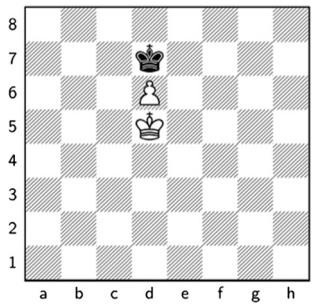

Diagram 1.

those of their opponent. When it is a player's turn, she attempts a move, which the umpire will declare to be 'legal' or 'illegal'. If the move is illegal, the player tries again; if it is legal, that move stands. Because information is imperfect, the player cannot fully control the situation.[6]

On the occasion of my first meeting with Lloyd Shapley, I held an International Master title in chess, and so I challenged him to a game of Kriegspiel. In order to test my readiness, Professor Shapley first presented a lesson in chess with imperfect information.

2. Lesson 1: How can you use your knowledge of game theory?

Shapley set up the position on Diagram 1 and explained that both White and Black are aware that this position is common knowledge. The ultimate purpose of the exercise, meanwhile, is to demonstrate how White wins with probability 1 in Kriegspiel. I was quite astonished, because any chess beginner can easily make a draw (for Black) by playing **Kd7 – d8 – d7**… and, at the right time, take the opposition on the eighth rank on **e8** and **c8**. However, it is not so straightforward in Kriegspiel. Let us see why. White plays **1. Ke5.** Black responds **1. … Kd8** and after **2. Ke6** Black does not know whether the White king is on **c6** or **e6**. Black has to guess and the right move, **2. … Ke8**, will be chosen with, for example, equal probability 1/2. Next, White will try to play **3. Ke6 – e7**. If this move is legal, then after **d6 – d7 – d8Q** White promotes the queen and wins.[7] Nevertheless if the umpire declares that move **3. Ke6 – e7** is illegal, then Black guessed correctly, and White is forced back to the initial Diagram 1.

White repeats the same procedure again and again. After n times the probability that Black always guessed correctly is $(1/2)^n$.[8] Can Black do something else? Yes, Black can play **1. … Ke8** and **2. … Kd8** trying to play "for sure." In this case White will never be able to play **3. Ke6 – e7** or **3. Kc6 – c7** and it might look like Black is always guessing correctly. Unfortunately, White wins with **3. d6 – d7** after **2. … Kd8** whether the White king is on **c6** or **e6**. Thus, White has to proceed as follows: carry out the king maneuver described above with probability $(\mathbf{1-\varepsilon})$ and play **d6 – d7** with probability $\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is very close to zero. Thus I learned that the endgame with a king and pawn against a lone king is winning almost surely.[9]

After lesson 1, Shapley presumed that I could checkmate a lone king with a rook and king even in Kriegspiel,[10] and so he complicated the task.

3. The infinite power of the rook[11]

Consider the following situation.

1. *The board is a quarter plane.*
2. *White's king and rook start as shown on Diagram 2.*
3. *Black places his king on any legal square, unknown to White.*
4. *White then plays to win with probability 1.*

Shapley, aware that I still lacked preparation, did not test me, and just showed the solution consisting of several steps.

[6] The rules of Kriegspiel are provided in Appendix C.

[7] We would assume that White can checkmate once the pawn is promoted to a queen, even though this task is not easy: White needs to avoid a stalemate or queen's capture in Kriegspiel.

[8] Note that usual chess "three-fold repetition" rule is not applicable in Kriegspiel (Rule 15 in Appendix C), so that White can repeat the described maneuver indefinitely.

[9] See Appendix A for more details.

[10] King and Rook vs King is the most widely studied ending in Kriegspiel. However, it is not simple by any means. See Boyce's (1981) and Ciancarini and Favini's (2009) algorithms for solving this problem.

[11] This problem was created by Lloyd Shapley in 1960. It is described *verbatim* from Shapley (1987). Shapley's description is in *italic*. See problem 12 in Appendix B.

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