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Q-statistics in dynamic speckle pattern analysis

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ABSTRACT

We introduce q statistics concepts to improve the performance of some methods based on the histogram to estimate dynamic speckle activity. It is shown that some improvements are obtained by choosing appropriate q values that have been empirically determined. The possibility of increasing the precision and diminishing the acquisition time are explored for a usual study case as is the drying of paint.

1. Introduction

In 1988 Tsallis [1] extended the domain of validity of standard thermodynamics and Boltzmann–Gibbs (B–G) statistical mechanics in order to cover a variety of anomalous systems. Systems involving long range interactions, long range microscopic memory in non-markovian stochastic processes, pure electron-plasma two dimensional turbulence, phonon–electron anomalous thermalization in ion-bombarded solids, solar neutrinos, etc. are some examples where the B–G theory presents difficulties. To overcome at least some of these problems, Tsallis' formalism proposed the extension of the concept of entropy by including a free parameter q to give origin to what is called *non extensive statistics*. This generalization proved to be very fruitful and, aside from thermodynamics, it was successfully applied to a wide variety of phenomena and to further generalizations including q parameter versions of several functions and operators (named *q deformed algebra*) such as *q Gaussian*, *q mean value*, *q exponential*, *q logarithm*, etc.

Some general properties of Tsallis entropy S_q are: S_q is positive, takes zero value for absolute certainty and increases monotonously with increasing uncertainty.

The generalization found interesting applications in widely different (some perhaps unexpected) fields. It has been applied to both theoretically well founded situations and experimental ones. A somewhat random view includes the order of words in a text, the citation of scientific papers, electroencephalography signals of epilepsy, to monitor brain injury after cardiac arrest, in financial markets, seismicity, atmospheric turbulence, gravitation, etc. An extended review of these subjects was presented in Refs. [2,3]. In image processing it has been applied to image registration, image thresholding, segmentation of cerebral tissue in multiple sclerosis magnetic resonance images, etc.

In optics, the study of the speckle patterns, due to their random nature, requires using statistical tools, the properties of which depend on the coherence of the incident light and the characteristics of the diffusing surface [4]. In the case of living samples and in some industrial processes, speckle patterns evolve in time and the dynamic speckles have different characteristics that can be used to obtain information on the phenomena participating in its origin [5].

In dynamic speckle metrology, where different statistic tools are required, some improvements in the measurements of activity, based on the histogram, could be expected by exploring different q values. However, the choice of the q value will be then dependent on the investigated phenomenon and the type of measurement.

In this work, we analyze some aspects of the statistical properties of dynamics speckle patterns using the Tsallis q formalism. We test q statistics adapted to that end to find the best suited values of the q parameter that optimize the application of several usual measurement algorithms in dynamic speckle. The improvements are, in some cases, rather subtle.

For $q=1$ the classic results are obtained; small (close to zero) q values emphasize the weight of rare events of the intensity histogram while high values of q emphasize the effect of frequent events. Since the effect of the variation of q is to change the balance between rare and frequent events, the improvements are in some cases are rather subtle.

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We present a brief description of the Tsallis q formalism and review some algorithms for the characterization of dynamic speckle patterns. Then, q mean value, q-entropy, q-standard deviation, LASCAq, a q variation of the inertia moment of the co-occurrence matrix and the Briers temporal contrast (BTC) are tested. We use a numerical simulation and a classic study of the drying of paint process. This is a well known process, the behavior of which is adequately characterized by dynamic speckle patterns [6]. The numerical simulation was used only in the technique (BTC) where experimental data could not be obtained and a well based theoretical basis already existed.

The q values in the 0, 1 interval were tested and it was found that some of these tools perform better when used with adequately chosen q values, as well as q values greater that 1 which we also tested. Values of q greater than 1 did not show any appreciable improvement in the tested cases. By adequately chosen values of q we mean that when those values are used, resolution is better or standard deviation is smaller, etc.

2. Theory

We work with dynamic speckle processes [7]. In the case where the sample shows a similar behavior all over its surface it is possible to study it with a single image obtained in a free propagation experimental device using objective speckles. This geometry provides a large amount of data describing simultaneously the same physical situation as contributions of the scattered light that are collected by every pixel of the detector. So, every pixel in the CCD camera provides a simultaneous measurement of the dynamics.

Conversely, if it is necessary to screen regions of the surface that show similar behavior, then an image forming configuration, named subjective speckle patterns, is required, and eventually many images might be needed to follow the time evolution of the intensity in each separate pixel to obtain significant statistics. As the processing with q statistics requires the use of the intensity histogram, for the results to be different from classical quantities ($q=1$) a higher number of samples is necessary. The sampling rate should then be faster than the explored phenomenon and it should be assumed stationary during the acquisition of the images.

Of course, the grabbing and processing of a single image is faster, and for fast phenomena this configuration might be the only adequate one. In what follows, we are going to refer them to the histogram of a spatial distribution obtained from a dynamic speckle pattern sample by free propagation.

2.1. q-mean value

To apply the q formalism in dynamic speckle techniques, following [2] we call *q-mean value* of a certain magnitude A:

$$\langle A\rangle_q \equiv \frac{1}{S}\sum_{i=1}^{W} p_i^q A_i \tag{1}$$

where p_i is the probability of the A_i value, q is called the parameter of non-extensivity and W is the number of possible values of the magnitude A. S is a normalization value defined as

$$S=\sum_{i=1}^{W} p_i^q \tag{2}$$

To calculate *q-mean value*, first the histogram of the intensity I is constructed. From the histogram the probabilities p_i are determined and the *q mean value* is calculated using Eq. (1) and using the intensity I as the magnitude A. This first step is a mapping of the ordinary normalized histogram $H(i)$ by raising every value of it to the q power and dividing by the normalizing factor. Fig. 1 shows an example of the result of this operation for three different q values: (a) $q=1$, (b) $q=0.05$ and (c) $q=3$. For $q=1$ the histogram is not modified and the classic mean value is obtained. For $q<1$ this operation has the effect of comparatively increasing the value of the probability of rare events. The modified histogram H_q (i) is then more even, thus resulting in an increase of the classical entropy. For $q>1$ the converse is true; frequent values are emphasized and classical entropy decreases.

The resulting $H_q(i)$ is then used for the calculation of q versions of other statistical measures used in dynamic speckle that are defined next.

2.2. q Variance

The *q-Variance* σ_q^2 is defined here as the q mean value of $(I-\langle I\rangle_q)^2$.

It is

$$\sigma^2{}_q=\left\langle \left(I-\langle I\rangle_q\right)^2\right\rangle_q \tag{3}$$

where I is the intensity.

2.3. LASCA (laser contrast speckle analysis)

LASCA [8] is an almost real time and non-scanning technique that uses the spatial first order statistics of time integrated speckle. It is used to build images to measure blood perfusion.

If intensity variations are relatively fast, finite integration time causes the (spatial) standard deviation $\sigma_{x,y}$ of the measured intensity I variations to diminish and so does the contrast defined as

$$C=\frac{\sigma_{x,y}}{\langle I\rangle} \tag{4}$$

where $\langle I\rangle$ is the spatial average of the intensity. This magnitude is a measure of the degree of blur exhibited by the diagram.

As C diminishes with increased activity and blur, an image constructed on this basis shows reversed contrast;with its active places appearing in dark regions and conversely. As a spatial standard deviation is required, the operation is calculated on spatial windows and involves some reduction in resolution. It is widely used in medicine applications [9].

LASCA with q values is defined as

$$C_q=\frac{\sigma_q}{\langle I\rangle_q} \tag{5}$$

where mean values and standard deviations are calculated using Eqs. (1) and (3), on time integrated speckle pattern spatial statistics.

In a spatial window for classical LASCA ($q=1$) there are few values (usually 3×3, 5×5, or 7×7 square pixels windows) as these windows are kept small to maintain the maximal spatial resolution. So, in general there will be few repetitions and the histogram will be very poor. There will be very few frequent values. Most of them will appear only once. If all the values are different (the most probable situation) then all the q powers of the p_i are the same for all q, so that in mean value calculations the value p_i appears as a common factor that can be taken out of the sum and cancels with the same value in the normalization denominator. All events are then equally frequent and the result is the same as with ordinary ($q=1$) LASCA. Something similar happens if all the values inside the window are the same.

Then, the result only differs from ordinary LASCA if some value appears more than once in the histogram and that variation is small unless q is very different from 1. Then, unless the window is big the result will be very similar to LASCA.

A big window is a serious limitation to image spatial resolution. It also requires that all the pixels inside it represent the same