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Simultaneous measurement of 3-D displacement components from circular grating moiré fringes: An experimental approach

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ABSTRACT

The moiré patterns formed by two overlapping circular gratings with slightly different periods were used for 2-D (in-plane) displacement measurement in a recently published paper. In this work, the application of the moiré patterns formed by such gratings is extended to 3-D displacement measurement. The 3-D measurement is based on the principle that the moiré pattern changes when the viewing direction is changed in the presence of out-of-plane displacement (z-component) between the two grating planes. Two different methods for the out-of-plane displacement measurement are proposed. The first method uses a single viewing direction (SVD) normal to the grating plane and co-axial with the circular grating. In this method, the moiré fringes move outwards for increasing z-displacement. This movement is represented by an offset shift in the polar transformed moiré fringes. The second method uses multiple viewing directions (MVD) whereby the moiré pattern changes when the viewing direction is shifted from the normal viewing direction to the off-normal viewing direction. The offset shift in the polar transformed moiré fringes in the SVD method and the absolute difference in the in-plane displacements extracted from successive moiré patterns in the MVD method were used to determine the z-displacement quantitatively. Experimental results show that the z-displacement component can be determined from both the SVD and MVD methods. However, the MVD method gives better accuracy and precision with an absolute mean error of 0.0533 ± 0.009 mm at 95% confidence compared to the SVD method which has an absolute mean error of 0.218 ± 0.065 mm.

1. Introduction

Although the moiré method was developed more than 100 years ago, it is still widely used for a variety of applications ranging from surface topography measurement [1–5], displacement and strain measurement [6–10], deformation measurement [11–13], vibration measurement [14,15], etc. The advantages of the moiré methods lie mainly in the simplicity of the experimental setup, reproducibility of the fringe patterns for specific relative displacements between the gratings pair, e.g. master and specimen grating, and the high amplification of displacement. Although linear gratings have been extensively used in moiré metrology, circular gratings are an attractive alternative for alignment and displacement measurements in 2-D (in-plane) because they form repeatable and unique moiré patterns for single axis in-plane displacements, i.e. either x- or y-displacement, as well as a combination of x- and y-displacements. The moiré patterns formed by overlapping circular gratings of slightly different pitches carry the information of eccentricity magnitude ε and direction φ of the two gratings as illustrated in Fig. 1. These two parameters (ε and φ) can be extracted quantitatively using the image analysis algorithm proposed by Yen and Ratnam [16,17] or using the intensity-based computation method developed by Park and Kim [18].

When there is a gap between the planes of the overlapping gratings, the circular gratings pair produces different moiré patterns when viewed from different angles relative to the normal of the grating plane. The change in the moiré pattern depends directly on the spacing between the grating planes. Thus, the moiré pattern is not only sensitive to in-plane displacement but also sensitive to out-of-plane displacement between the two gratings. This phenomenon was used by Lay and Chen [19] to detect the rotation angle of a head-mounted display. This was achieved by aligning a pair of identical gratings co-axially but with a small gap in-between the gratings. The moiré pattern changed when the viewing direction was off-normal relative to the gratings plane. The moiré pattern formed in the off-normal viewing direction was then analyzed to determine the rotation angle to a very high accuracy with an error rate of only 0.005%. However, there was no attempt to measure the absolute 2-D (x- and y-) displacement components or the z-displacement between the grating planes. To date, a quantitative method for

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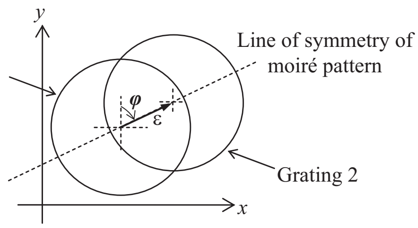

Fig. 1. Eccentricity magnitude ε and direction φ of superimposed circular gratings (only a single circle is shown on each grating).

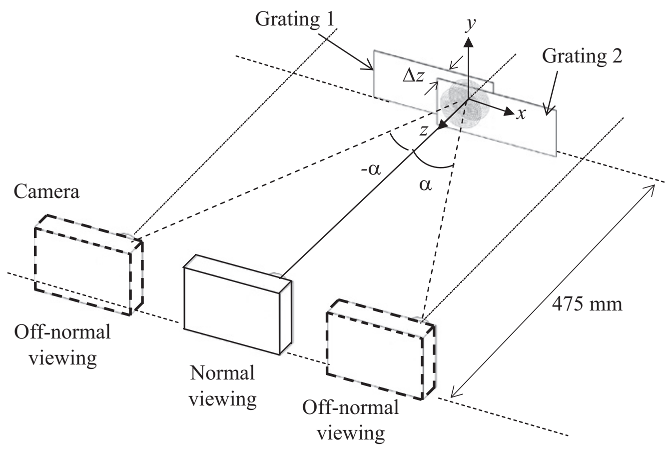

Fig. 2. Viewing directions relative to the grating plane.

the simultaneous 3-D displacement measurement using the circular gratings has not been reported.

2. Principle of the proposed method

Two types of viewing directions are used in this study, namely the normal viewing and off-normal viewing directions. Fig. 2 schematically illustrates the viewing directions relative to the grating plane. Normal viewing is in the direction normal to the grating plane, located co-axially with the circular pattern in Grating 1 and is in-line with the z-axis, i.e. $\alpha=0^\circ$. On the other hand, off-normal viewing refers to the viewing direction that forms an angle α ($\neq 0^\circ$) to the normal viewing direction. In the experimental setup both viewing directions are located in the x–z plane.

Fig. 3(a)–(c) shows some sample moiré patterns for different z-displacements when viewed from various viewing directions. These moiré patterns were generated using the commercial gratings developed by Bill Harvey Associates (UK) Ltd. Each grating has 34 circles with a pitch of 0.55 mm and 0.50 mm. The line width of each circle is 0.23 mm. This gratings set is designed to produce three concentric ring-shape moiré patterns when they are overlapped concentrically. In each case the in-plane displacement components are both zero. The pattern in the middle of each set was viewed normal to the grating plane ($\alpha=0^\circ$) while the patterns in the right and left were viewed at $\alpha=+3.6^\circ$ and $\alpha=-3.6^\circ$ off the normal direction, respectively. Two types of changes can be observed in the moiré patterns when one of the gratings is displaced in the z-direction. The first observation is the change in the moiré patterns captured at viewing direction $\alpha=0^\circ$ in each image set. At the viewing direction $\alpha=0^\circ$ the moiré fringes expand gradually when the z-displacement is increased from 1.0 mm to 5.0 mm. This can be seen more clearly by observing the outer-most fringe in Fig. 3(a)–(c) for $\alpha=0^\circ$. The second observation is that the moiré pattern changes when the viewing direction is shifted from $\alpha=0^\circ$ to $\alpha=3.6^\circ$ or $\alpha=-3.6^\circ$ for the same z-displacement. The change is more obvious for higher values of z-displacement. The moiré patterns viewed from the off-center position carry '2-D displacement' information similar to that observed from the normal viewing direction in the presence of in-plane displacements. For the same off-center viewing direction a greater '2-D displacement' occurs when the z-displacement is increased from 1.0 mm to 5.0 mm.

Fig. 4(a) shows the moiré patterns formed by different 3-D displacement inputs (x, y, z) while the corresponding transformed patterns are shown in Fig. 4(b). These patterns were viewed from the direction $\alpha=0^\circ$. The expansion in the moiré pattern for increasing z-displacement can be seen more clearly when the moiré patterns are transformed into the polar coordinate plane (r–θ) as shown in Fig. 4(b). The transformed moiré patterns shift in one common direction (downwards in the diagram) for increasing z-displacements. Using image processing techniques, it is possible to extract quantitative information from the transformed moiré patterns that reflect the change and relate it to the z-displacement values.

For the off-normal viewing direction the presence of the z-displacement component between the two overlapping gratings causes a shift v in the image plane as seen in Fig. 5. M and N are the centers of the fixed grating and moveable grating, respectively, while M' and N' are the images of these points on the image plane. The relationship between the z-displacement component and the shift v is given by:

$$z=\frac{(l-h)v}{(l\tan\alpha-s)+v}. \tag{1}$$

By knowing l, α, h, s and v, the z-displacement between the gratings can be determined from Eq. (1). The parameters l, α and s are the external dimensions that can be determined directly from the experimental setup while h is the distance between the camera lens nodal point and the sensor plane of the camera, which cannot be determined easily. The shift v is the in-plane displacement between the gratings represented by the moiré pattern and can be determined using the 2-D measurement method developed previously [16,17] and, hence, the z-displacement component can be found using Eq. (1). However, the direct determination of z-displacement from Eq. (1) is prone to errors due to the errors in the determination of l, α, h and s in the equation. The error in the determination of the in-plane displacement components will also introduce more errors in the z-displacement component. Thus, a calibration method based on an experimental approach is proposed to obtain the z-component displacement with a higher accuracy.

Two methods are proposed to analyze the moiré patterns formed by the 3-D displaced circular gratings and to determine the 3-D displacement components. The first method is named the single viewing direction (SVD) method that analyzes the change of the moiré pattern observed at the fixed viewing direction of $\alpha=0^\circ$ when the z-displacement is changed. The second method is called the multiple viewing directions (MVD) method that relates the absolute displacement change represented by the moiré patterns at normal viewing direction ($\alpha=0^\circ$) and a predefined off-normal viewing direction ($\alpha\neq 0^\circ$).

3. Methodology

3.1. 2-D displacement determination from moiré patterns

The moiré patterns for different z-displacement values observed from the various viewing directions carry information similar to that in the 2-D displacement measurement. Thus, analysis of the