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A comparative study of pricing approaches for longevity instruments

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ABSTRACT

The presence of systematic risk in mortality forecasts, known as longevity risk, has called for the introduction of longevity instruments and their market development. Management of longevity risk has been an ongoing issue for insurance companies and pension funds who offer products with payout depending on the lifetime of policyholders. One of the major difficulties in pricing longevity instruments is the determination of a longevity risk-premium. This problem arises from the fact that the longevity market is illiquid and is considered to be incomplete. In this paper we provide an insight to the study of several pricing approaches for longevity instruments that have been proposed in the literature. To account for parameter uncertainty in mortality forecasts and longevity instruments pricing, our analysis hinges on a Bayesian state-space mortality model. The sampling-based Bayesian approach allows us to obtain a distribution of the longevity risk-premium, thus providing an alternative perspective in analyzing the pricing methods. We also discussed the advantages and disadvantages of the considered pricing approaches.

1. Introduction

The provision of retirement income products such as the life annuity, the deferred annuity and the variable annuity plays an important role in dealing with the increased financial burden for societies caused by the low adequacy of government-sponsored pension schemes. A common feature of retirement income products is to provide a stable stream of income during an individual's retirement phase. Pricing and risk management of these types of policies relies crucially on the accuracy of mortality forecasts. Recently, mortality risk, which is the risk resulting from the deviation of mortality rates from its forecasted value, has been exacerbated by the systematic risk associated with consistent mis-estimation of life expectancy (see Deng et al., 2012), this will become more salient in an ageing population with a high dependency ratio (see Alonso-García et al., 2017), because of this it has called for annuity providers and insurers to tackle the problem of longevity risk, as well as the introduction of the capital markets as a new market participant in longevity risk management.

One common approach to manage longevity risk in annuity portfolios is to set up capital reserves such as those required by Solvency II to absorb any extra loss caused by an underestimation of longevity improvement (Olivieri and Pitacco, 2008; Blackburn et al., 2017). Whereas, another important approach is to use longevity instruments, or longevity derivatives, to transfer excessive longevity risk undertaken by annuity providers to capital markets. Various types of longevity instruments have been traded in practice and proposed in the literature. Coughlan (2014) reports that nearly thirty longevity swaps have been executed by defined benefit pension plans in the UK from 2009–2014; several innovative longevity hedging instruments were transacted during 2010–2014. Other types of products that have been proposed include the EIB–BNP longevity bond and Swiss Re Kortis longevity bond which are both studied in Cairns et al. (2006a) and Hunt and Blake (2015) respectively. Standardized longevity hedging instruments,[1] such as q-forwards, S-forwards and longevity swaps are considered in Ngai and Sherris (2011), Fung et al. (2014) and Biffis et al. (2016).

One of the major obstacles in developing longevity markets for standardized instruments is the presence of basis risk,[2] that is the risk that the mortality experience of an annuity portfolio may differ from the underlying mortality index of a standardized longevity hedging instrument. The forecast of mortality then plays a crucial role in predicting the mortality experience, and for the past few years there have been many mortality models that have been developed and used in the context of pricing. One example is by Bauer et al. (2010b) who uses the forward mortality framework to analyze various pricing methods (see also Cairns et al., 2008; Blackburn and Sherris, 2013).

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[1] Life and Longevity Markets Association (LLMA); http://www.llma.org/.

[2] Quantification and analysis of basis risk is studied in Li and Hardy (2011), Chan et al. (2016) and De Rosa et al. (2016).

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As discussed in Czado et al. (2005) and Koissi et al. (2006), parameter uncertainty is an important aspect in mortality model estimation and forecasting. For this reason we adopt the Bayesian state-space framework that treats stochastic mortality models as state-space models, see Pedroza (2006), Kogure and Kurachi (2010a) and Fung et al. (2017). Specifically, we consider the Cairns–Blake–Dowd (CBD) model under the Bayesian state-space methodology and employ it as the benchmark mortality model to analyze the pricing approaches. The CBD model has the advantage that no identification constraint is required unlike Lee–Carter (LC) type models, and the model is designed for capturing mature-age mortality dynamics (see Cairns et al., 2011; Maccheroni and Nocito, 2017) which is particularly suitable for pricing longevity instruments. Moreover, being a 2-factor model the CBD model provides extra flexibility in capturing the underlying dynamics of mature-age mortality rates while preserving simplicity and robustness for modeling mortality across various countries, see Dowd et al. (2010b). A state-space approach to the CBD model in a frequentist setting is considered in Liu and Li (2016a, b). In mortality modeling context dataset is often relatively small compared to the number of parameters that needs to be estimated thus capturing parameter uncertainty is of particular importance. Bayesian statistics offers a rigorous framework in which one can incorporate parameter uncertainty by assuming parameters are random variables. Moreover, expert opinion can be taken into account by specifying prior distributions for the parameters, which are then combined with observed data (or evidence) to produce the posterior distributions for the parameters. In addition, thanks to the advancement of sampling methods such as Markov chain Monte Carlo (MCMC) (see Carter and Kohn, 1994), more realistic and sophisticated modeling assumptions can be made while estimation and forecasting can still be performed efficiently under the Bayesian framework.

Another major obstacle is to determine an adequate pricing framework for longevity derivatives, as the longevity market is illiquid and considered to be incomplete. According to no-arbitrage pricing theory, a market is incomplete implies that one cannot uniquely determine the price of a derivative (Bjork, 2009). As a result, further assumption, for example through economic or partial hedging argument, is required to decide the derivative price under a particular pricing framework. A number of approaches have been proposed in the literature to deal with the primary issue in an incomplete market. In this paper we conduct a comparative study of these pricing approaches in the longevity risk context, with the help of a Bayesian framework by capturing parameter uncertainty in the model estimation stage. This is particularly important in an incomplete market, due to the fact that there usually exists a lack of publicly available prices which causes problems to investors who are conducting risk evaluation. The lack of prices of liquid instruments implies that model risk plays a particularly important role in pricing where parameter uncertainty is a significant component. In addition, alternative markets which are illiquid are also able to take advantage of this paper to conduct pricing and risk analysis under our proposed framework.

We consider four pricing approaches consisting of (1) the Risk-Neutral pricing method from no-arbitrage pricing theory developed in financial economics (Cairns et al., 2006a; Bjork, 2009); (2) the Wang transform method proposed to price financial and insurance contracts (Wang, 2002a; Denuit et al., 2007; Cox et al., 2010); (3) the Canonical valuation method where the risk-adjusted measure is found by minimizing the so-called Kullback–Leibler Information Criterion (Stutzer, 1996; Kogure and Kurachi, 2010b; Li, 2010; Li and Ng, 2011) and (4) an economic approach known as Tatonnement method relying on the utility functions of buying and selling agents to determine the price of a security (Walras and Jaffé, 1954; Zhou et al., 2015). Although this is by no means an exhaustive list of pricing approaches, the four methods considered in this paper covers of a wide range of pricing approaches, ranging from a financial aspect; model and model free methods; and lastly an economic based approach.[3] One method that was not considered but is closely related to the method of the Tatonnement approach is the utility indifference pricing method proposed by Hodges and Neuberger (1989). Eichler et al. (2017), applied the utility indifference pricing method in catastrophe bonds; Møller (2003) and Dahl and Møller (2006) applied it in a mean–variance context, and Alonso-García and Devolder (2016) applied it to calculating the guarantees on population wages risk.[4]

Pricing of longevity securities is studied in Bauer et al. (2010b) where the Wang transform method and the instantaneous Sharpe ratio approach (Bayraktar et al., 2009) are considered. The instantaneous Sharpe ratio approach is equivalent to a change of the real-world measure to a risk-adjusted measure by assuming a constant market price of risk process (Bauer et al., 2010b). In this paper we do not consider the instantaneous Sharpe ratio approach. For further discussion of this approach, see Bayraktar and Young (2008) and Bayraktar et al. (2009).

The paper contributes to the literature in three aspects. First, a MCMC-based Bayesian inference for the CBD model is developed under the state-space framework. We discuss in detail the estimation of the correlation matrix for the hidden state dynamics of the CBD model. Second, using the posterior samples we obtain the distribution of the longevity risk-premium by calibrating the model to a traded longevity instrument, this provides an alternative perspective in analyzing the pricing methods. Similarly we show that parameters of the utility functions used in the Tatonnement approach can be obtained by a model calibration procedure. Third, we systematically analyze the pricing methods considered and comment on the advantages and disadvantages of these different approaches. It is important to note that our contribution is to compare pricing methods without relying on real longevity-linked securities market data and that the premiums obtained are not representing the actual risk premiums in the market.

The paper is organized as follows. Section 2 develops a Bayesian state-space framework for the CBD model that will be used as the benchmark model for analyzing different pricing approaches. Section 3 discusses longevity instruments and investigate several pricing approaches that can be used to price these instruments. A comparative study of the considered pricing approaches is reported in Section 4. Section 5 concludes.

2. Stochastic mortality modeling

A key aspect in pricing longevity instruments is the assumption of the underlying mortality model. We utilize the state-space framework for stochastic mortality modeling to analyze different pricing methods for longevity instruments. The choice here reflects our belief that parameter uncertainty should be taken into account for model estimation and mortality forecasts. The state-space approach provides a particularly accommodating environment where Bayesian inference for dynamic mortality models can be carried out efficiently based on MCMC method (Pedroza, 2006; Kogure and Kurachi, 2010a; Fung et al., 2017). A Bayesian state-space formulation, estimation and forecasting for the CBD mortality model is developed in this section.

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[3] Other methods such as: Cvitanić et al. (1999) focus on replicating a European type option under an incomplete market driven by two Brownian motions, one for the underlying process and the other for the volatility coefficient in the underlying process; in Artzner et al. (1999) they focus on the theory behind a coherent risk measure under an incomplete market; whereas Cochrane and Saa-Requejo (2000) prices index based options based on an economic approach which relaxes the idea of market completeness, they assumed bounds on the Bank account process and that investors aim to buy assets with high sharpe ratios.

[4] We thank the anonymous reviewer for their suggestion to the aforementioned papers.