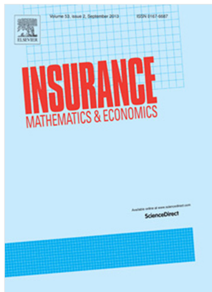
Accepted Manuscript



Compound unimodal distributions for insurance losses

Antonio Punzo, Luca Bagnato, Antonello Maruotti

PII: S0167-6687(17)30207-X
DOI: https://doi.org/10.1016/j.insmatheco.2017.10.007
Reference: INSUMA 2415

To appear in: *Insurance: Mathematics and Economics*

Received date : April 2017
Revised date : October 2017
Accepted date : 22 October 2017

Please cite this article as: Punzo A., Bagnato L., Maruotti A., Compound unimodal distributions for insurance losses. *Insurance: Mathematics and Economics* (2017), https://doi.org/10.1016/j.insmatheco.2017.10.007

Compound unimodal distributions for insurance losses

Antonio Punzo[a], Luca Bagnato[b], Antonello Maruotti[c,*]

[a] *Dipartimento di Economia e Impresa, Università di Catania, Catania*
[b] *Dipartimento di Discipline Matematiche, Finanza Matematica e Econometria, Università Cattolica del Sacro Cuore, Milano*
[c] *Dipartimento di Scienze Economiche, Politiche e delle Lingue Moderne, Libera Università Maria Ss. Assunta, Roma*

Abstract

The distribution of insurance losses has a positive support and is often unimodal hump-shaped, right-skewed and with heavy tails. In this work, we introduce a 3-parameter compound model to account for all these peculiarities. As conditional distribution, we consider a 2-parameter unimodal hump-shaped distribution with positive support, parametrized with respect to the mode and to another variability-related parameter. The compound is performed by scaling the latter parameter by a convenient mixing distribution taking values on all or part of the positive real line and depending on a single parameter governing the tail behavior of the resulting compound distribution. Although any 2-parameter distribution can be considered to derive its compound version in our framework, for illustrative purposes we consider the unimodal gamma, the lognormal, and the inverse Gaussian. They are also used as mixing distributions; this guarantees that the un-compound distribution is nested in the compound model. A family of nine models arises by combining these choices. These models are applied on three famous insurance loss datasets and compared with several standard distributions used in the actuarial literature. Comparison is made in terms of goodness-of-fit and through an analysis of the commonly used risk measures.

Keywords: Mode, Positive support, Normal scale mixture, Insurance losses, Risk measures, Heavy tailed distributions.
JEL codes: C46, C51, C52

1. Introduction

Insurance loss data are positive (Klugman et al., 2012) and their distribution is often unimodal hump-shaped (Cooray and Ananda, 2005), right-skewed (Lane, 2000), and with heavy tails (Ibragimov et al., 2015). Though many parametric unimodal distributions have been used in the actuarial literature for modeling these data (Klugman et al., 2012), their peculiarities call for more flexible models (Ahn et al., 2012).

Right-skewness may be accommodated by skewed distributions (Lane, 2000 and Bernardi et al., 2012). In this class, Vernic (2006), Bolancé et al. (2008), Adcock et al. (2015), and Kazemi and Noorizadeh (2015) identify the skew-normal as a promising model. However, using the skew-normal distribution is in principle appropriate when the support is the whole real line, while it is not adequate if the support is the positive real line as it causes *boundary bias*, that is, allocation of probability mass outside the theoretical support. A possible solution consists in considering transformations, for example the logarithm, so as to make the support the whole real line and then fitting the skew-normal distribution. Although such a treatment is very simple to use, the transformed variable may become more difficult to be interpreted (Bagnato and Punzo, 2013). Instead of applying transformations, there is a growing interest in proposing

*Corresponding author. Dipartimento di Scienze Economiche, Politiche e delle Lingue Moderne, Libera Università Maria Ss. Assunta, Via Pompeo Magno 22, 00192, Roma, Italy. Phone: +39 06 68422487
Email addresses: antonio.punzo@unict.it (Antonio Punzo), luca.bagnato@unicatt.it (Luca Bagnato), a.maruotti@lumsa.it (Antonello Maruotti)