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Robust optimal investment strategy for an AAM of DC pension plans with stochastic interest rate and stochastic volatility

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ABSTRACT

In this paper, we investigate a robust optimal investment problem for an ambiguity-averse member (AAM) of defined contribution (DC) pension plans with stochastic interest rate and stochastic volatility. The AAM has access to a risk-free asset, a bond and a stock in a financial market. We assume that the interest rate is described by an affine model, which includes the Cox–Ingersoll–Ross model and the Vasicek model as special cases, while the stock price is driven by the Heston's stochastic volatility model. Moreover, the AAM has different levels of ambiguity aversion about the diffusion parts of the interest rate and the stock's price and volatility. She attempts to maximize the expected power utility of her terminal wealth under the worst-case scenario. By applying the stochastic dynamic programming approach, we derive a robust optimal investment strategy and the corresponding value function explicitly, and subsequently two special cases are discussed. Finally, a numerical example is presented to illustrate the impact of model parameters on the robust optimal investment strategy and to explain the economic meaning of our theoretical results. The numerical example shows that the AAM's ambiguity aversion levels about the interest rate and the stock's price and volatility have different impacts on the proportions invested in the risky assets, and that ignoring model uncertainty always incurs utility losses for the AAM.

1. Introduction

Recent decades have witnessed the longer lifespans and the lower fertility rate, which have highlighted the importance of pension management. As one of the main types of pension plans, the defined contribution (DC) pension plan has the advantage of relieving the pressure faced by the social security system, which transfers the longevity and financial risks from the sponsor to the member. This can be attributed to the fact that in a DC pension plan, the contributions are predetermined and the benefits are adjusted depending on the return of the pension plan's portfolio. Therefore the investment problem of DC pension plans has attracted significant attention in the financial and the actuarial literature. There are many studies considering the optimal investment strategy before retirement to maximize the expected utility from terminal wealth, for example, Devolder et al. (2003), Cairns et al. (2006) and Korn et al. (2011). Since the investment of a pension plan usually lasts for a long period, generally 20–40 years, it is crucial to take the risk of interest rate into account. Boulier et al. (2001) incorporate the Vasicek interest rate model and a minimum guarantee at retirement into a DC pension investment problem. Deelstra et al. (2003) extend the model of Boulier et al. (2001) to an affine interest rate model including the Vasicek model and apply the martingale method perfectly. Gao (2008) further applies the Legendre transformation and dual theory to solve the same portfolio problem of a DC pension plan with a logarithm utility function.

Despite the vast literature on the optimal investment problem for DC pension plans, the majority of studies assume that the stock price is modeled by a geometric Brownian motion, i.e., the volatility of the stock price is a constant or a deterministic function. In reality, however, the stock price may have different characteristics. There are many empirical studies supporting the existence of stochastic volatility (SV) for the stock price (see, e.g., French et al., 1987; Pagan and Schwert, 1989; Hobson and Rogers, 1998). Gao (2009) studies the optimal investment strategy for a DC pension plan in which the stock price is modeled by the constant elasticity of variance (CEV) model. Apart from the CEV model known as a local volatility model, the Heston's SV model is also an excellent tool to describe the stock price. Actually, the Heston's SV model has become one of standard approaches in the pricing of financial derivatives, see Liu and Pan (2003) and Sepp (2008). Recently, the Heston's SV model is also widely used in the field of insurance. Li

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et al. (2012), Zhao et al. (2013) and Li et al. (2016) all adopt the Heston's SV model to describe the stock price and obtain the optimal investment and reinsurance strategy under mean–variance criterion or the criterion of utility maximization. However, the investment problem for a DC pension plan under the Heston's SV model is just introduced by Guan and Liang (2014). They also consider the risk of interest rate and obtain an explicit solution by applying the stochastic dynamic programming method.

In traditional settings of DC pension investment problems, decision-makers are assumed to know exactly the true probability measures used for these investment problems. However, in many situations decision-makers are uncertain about the true model, because, for example, the parameters (especially the drift parameters) are hard to estimate with precision (Merton, 1980; Cochrane, 1997). Therefore, it is fair to say that any particular probability measure used to describe the model would be subject to a considerable degree of model misspecification. This type of uncertainty caused by the lack of information about the probability measure is also referred to as ambiguity, which is apparently different from risk where the model is characterized by a single probability measure (Knight, 1921). Moreover, experimental studies demonstrate that individual investors not only display aversion to risk but also display aversion to ambiguity (Ellsberg, 1961; Bossaerts et al., 2010). These facts motivate more and more scholars to study the impact of ambiguity on portfolio choice and asset pricing.

To deal with ambiguity aversion, Anderson et al. (2003) propose a robust control approach in a continuous-time framework. They assume that the decision-maker has a specific probability measure, but she does not trust the probability measure. Therefore, she only treats the specific measure as a reference measure, and takes into account a set of alternative measures that is statistically difficult to distinguish from the reference measure. The gap between the reference measure and an alternative measure is constrained by the relative entropy, which acts as a penalty term in the optimization procedure. This penalty captures the decision-maker's ambiguity aversion about the reference measure. Maenhout (2004) improves the robust control approach by proposing "homothetic robustness", which allows him to obtain closed-form solutions in a dynamic portfolio and consumption problem. Maenhout (2006) further investigates an optimal portfolio choice problem with stochastic investment opportunities under ambiguity, and presents a method to calculate detection-error probabilities. In line with Maenhout (2004, 2006) and Flor and Larsen (2014) incorporate ambiguity into an optimal investment problem with stochastic interest rates described by the Vasicek model, and the investor is assumed to be ambiguous about the expected returns of both bonds and stocks. Subsequently, Escobar et al. (2015) investigate an optimal portfolio problem under stochastic volatility for an ambiguity-averse investor who is allowed to trade in stock and derivatives. In addition, Yi et al. (2013) and Zheng et al. (2016) study optimal reinsurance–investment problems for ambiguity-averse insurers under stochastic volatility using the Heston's SV model and the CEV model, respectively.

In practice, there is still no agreement on which probability measure should be used for the optimal investment problem of DC pension plans. So, ambiguity does exist and it is quite significant to incorporate ambiguity into the problem. To the best of our knowledge, there is little published literature on DC pension plans that takes ambiguity into account. Therefore, in this paper, we introduce ambiguity, stochastic interest rate and stochastic volatility together into a DC pension investment problem. We assume that a representative DC plan member is ambiguity averse. In our model, the ambiguity-averse member (AAM) has access to a financial market consisting of a risk-free asset, a bond and a stock and controls her account's investment strategy. Specifically, the stochastic interest rate follows an affine model which includes the Cox–Ingersoll–Ross (CIR) model and the Vasicek model, meanwhile, the stock price is described by the Heston's SV model. The salary is also stochastic and driven by a geometric Brownian motion. Following Uppal and Wang (2003), the AAM is assumed to have different levels of ambiguity aversion about the interest rate and the stock's price and volatility. Based on the above settings, we establish a robust optimal investment problem for the AAM with power utility. Using the stochastic dynamic programming approach, we derive analytical expressions of the robust optimal investment strategy, as well as the corresponding value function. Furthermore, two special cases of our model are discussed and the corresponding results are provided. Finally, the economic implications of our theoretical results and the utility loss from ignoring ambiguity are analyzed by using a numerical example. The main contribution of this paper is threefold. (i) In an optimal investment problem for a DC pension plan, we introduce ambiguity, stochastic interest rate and stochastic volatility simultaneously. (ii) We show the impacts of the AAM's ambiguity aversion levels about the interest rate and the stock's price and volatility on the robust optimal investment strategy respectively, and we also study the influence of the salary on the robust optimal investment strategy. (iii) We find that an AAM who does not consider the impacts of ambiguity (especially ambiguity about the stock dynamics) on the optimal strategy suffers severe utility loss.

The rest of this paper is organized as follows. Section 2 describes the assumptions and the formulation of the model. Section 3 drives explicit expressions of the robust optimal investment strategy and the corresponding value function under some technical conditions. Section 4 provides two special cases of our model. Section 5 presents a numerical example to illustrate the effects of model parameters on the robust investment strategy and the utility loss from ignoring ambiguity. Section 6 concludes this paper.

2. General formulation

In this paper, we consider a robust optimal investment problem for a DC pension plan. We assume that trading in the financial market is continuous, no transaction costs or taxes are involved, and short selling is permitted. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with a filtration $\{\mathcal{F}_t\}_{0\le t\le T}$ which satisfies the usual conditions, i.e., $\{\mathcal{F}_t\}_{0\le t\le T}$ is right-continuous and $\mathbb{P}$-complete, where $\mathcal{F}_t$ denotes the information set available until time t. Moreover, we assume that all the stochastic processes described below are well-defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and adapted to $\{\mathcal{F}_t\}_{0\le t\le T}$.

2.1. Financial market and salary

The financial market consists of three assets: a risk-free asset, a rolling bond and a stock. The price of the risk-free asset satisfies the ordinary differential equation

$$\frac{dS_0(t)}{S_0(t)} = r(t)dt, \quad S_0(0) = s_0, \tag{1}$$

where $r(t)$ is the instantaneous interest rate. In general, the investment of the pension fund involves a relatively long period, and hence the constant interest rate may not be rational. Therefore, we assume that the instantaneous interest rate is described by a stochastic differential equation

$$dr(t) = (a - br(t))dt - \sqrt{k_1 r(t) + k_2}dW_r(t), \quad r(0) = r_0, \tag{2}$$