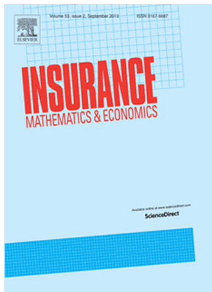

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Large deviations for risk measures in finite mixture models*

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Abstract

Due to their heterogeneity, insurance risks can be properly described as a mixture of different fixed models, where the weights assigned to each model may be estimated empirically from a sample of available data. If a risk measure is evaluated on the estimated mixture instead of the (unknown) true one, then it is important to investigate the committed error. In this paper we study the asymptotic behaviour of estimated risk measures, as the data sample size tends to infinity, in the fashion of large deviations. We obtain large deviation results by applying the contraction principle, and the rate functions are given by a suitable variational formula; explicit expressions are available for mixtures of two models. Finally, our results are applied to the most common risk measures, namely the quantiles, the Expected Shortfall and the shortfall risk measure.

AMS Subject Classification. Primary: 60F10, 91B30. Secondary: 62B10, 62D05.
Keywords: contraction principle, Lagrange multipliers, quantile, entropic risk measure, relative entropy.

1 Introduction

Quantitative risk management for financial and insurance companies requires the modelling of financial positions in terms of random variables on a suitable probability space; in mathematical terms, this corresponds to identifying a probability law (model) μ on the real line that describes as accurately as possible, the random behaviour of the position. Model risk, that arises from the uncertainty about the model to adopt, has been largely discussed in various area of the literature, because it may impact substantially companies decision making and performance. We can distinguish three main approaches to deal with model uncertainty: 1) the model is not specified but directly extrapolated from data via the empirical distribution; 2) a model is selected and its parameters are estimated from data (e.g. using Maximum Likelihood Estimation); 3) a class of candidate models is considered (for instance models suggested by expert opinion) and then one or an average of them is applied. The latter approach is probably the most common one and includes for instance: the worst-case approach proposed by Gilboa and Schmeidler (1989) in the theory of utility maximization, where the chosen model is the one providing the most adverse outcome; the Bayesian model averaging approach, developed by Raftery et al. (1997) where (posterior) weights are calculated for each model considering both information arising from data and prior beliefs; the highest posterior approach, where the selected model is the one most favourable according to the posterior weights. Cairns (2000) provided a general framework for dealing with model and

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