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Optimal robust reinsurance-investment strategies for insurers with mean reversion and mispricing☆

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HIGHLIGHTS

- An optimal robust reinsurance-investment problem for insurers is studied.
- Mean reversion, mispricing and model ambiguity are incorporated in our model.
- Optimal reinsurance-investment strategy and optimal value function are derived.
- Dynamic programming approach is used and two special cases are discussed.
- Numerical examples are presented to illustrate the impact of some parameters.

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ABSTRACT

This paper considers how to optimize reinsurance and investment decisions for an insurer who has aversion to model ambiguity, who wants to take into consideration time-varying investment conditions via mean reverting models, and who wants to take advantage of statistical arbitrage opportunities afforded by mispricing of stocks. We work under a complex realistic environment: The surplus process is described by a jump–diffusion model and the financial market contains a market index, a risk-free asset, and a pair of mispriced stocks, where the expected return rate of the stocks and the mispricing follow mean reverting stochastic processes which take into account liquidity constraints. The insurer is allowed to purchase reinsurance and to invest in the financial market. We formulate an optimal robust reinsurance-investment problem under the assumption that the insurer is ambiguity-averse to the uncertainty from the financial market and to the uncertainty of the insured's claims. Ambiguity aversion is an aversion to the uncertainty taken by making investment decisions based on a misspecified model. By employing the dynamic programming approach, we derive explicit formulae for the optimal robust reinsurance-investment strategy and the optimal value function. Numerical examples are presented to illustrate the impact of some parameters on the optimal strategy and on utility loss functions. Among our various practical findings and recommendations, we find that strengthened market liquidity significantly increases the demand for hedging from the mispriced market, to take advantage of the statistical arbitrage it affords.
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1. Introduction

Investments in financial markets provide an important mean for any insurer to increase profits from the surplus process, while reinsurance is key to helping the insurer avoid or transfer excessive risk. Investments and reinsurance are therefore rightfully attracting increasing attention from a growing number of insurance mathematics scholars, and are becoming a popular topic in the actuarial literature. For example, Schmidli (2002) considered the optimal reinsurance-investment problem of minimizing ruin probability; Bäuerle (2005) studied the optimal reinsurance problem with a mean–variance criterion; Bai and Guo (2008) focused on

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the optimal reinsurance-investment problem by maximizing exponential utility with a no-shorting constraint; Zeng and Li (2011) considered the optimal time-consistent reinsurance-investment problem with mean variance criterion; Gu et al. (2012) investigated the optimal excess-of-loss reinsurance and investment problem with a Constant-Elasticity-of-Variance model.

However, most papers listed above assume the risky asset's appreciation rate is a constant or a deterministic function. This goes against empirical stock market data as widely understood and noted by many authors (see for instance Merton (1980) and Rapach and Zhou (2013)): in reality, appreciation rates are not constant, and more specifically, mean rates of return are more appropriately modeled as being stochastic processes; they are thus often referred to as *stochastic risk premia*. Typically, stochastic risk premium processes are described by mean-reverting (MR) models, which are given in the empirical literature on real market, see Chapter 20 in Cochrane (2001) and Rapach and Zhou (2013); they play an important role in the portfolio by providing for the possibility of time-varying investment opportunities. Thus, a MR risk premium is seen as a valuable feature of the risky asset's price. Baev and Bondarev (2007), who discussed the ruin probability of an insurer, studied the optimal investment problem for a risky model in which the stock price has a stochastic risk premium given by an Ornstein–Uhlenbeck (OU) process, which is a special type of MR process. The OU process has the advantage of being amenable to explicit analyses in many cases because it is Gaussian. Liang et al. (2011) aimed to maximize the expected exponential utility of the terminal wealth where again the instantaneous rate of the stock's return follows an OU process; they obtained the optimal reinsurance and investment strategies. Gu et al. (2013) studied the optimal DC pension plan under an OU model. Recently, Yi et al. (2015a) studied dynamic portfolio selection with mean reversion describing the stochastic risk premium, under additional current specificities of Chinese and Hong Kong stock markets.

In this paper, we incorporate ambiguity aversion to study the optimal robust reinsurance-investment problem, where the stock's stochastic risk premium follows a MR process. At the moment, the use of MR risk premia is understudied in the implementation of robust optimization. Going back to the original work of Anderson et al. (2000) discussed below, and the seminar work of Maenhout (2004) on homothetic robustness, we know that robust optimization is helpful to account for the fact that drift coefficients are very difficult to pin down. Also, we describe several other papers above and below which use robustness this way, to give some flexibility on the inability to estimate a constant drift coefficient reliably. However, as stated above, if the real problem is that mean rates of return are difficult to estimate because they are actually stochastic, as in the case of stochastic risk premia, allowing for an uncertain constant drift parameter in an optimal investment problem does not capture the time-varying aspect of investment conditions in the models' specifications. In this sense, using a MR process for the risk premium is crucial for many stock markets where such a model feature is widely accepted (see Poterba and Summers (1988), Fouque et al. (2000), and Mukherji (2011) for empirical evidence). The paper by Yi et al. (2015a) about dynamic portfolio selection with MR stochastic risk premium takes this approach and combines it with an acceptance of modeling uncertainty, including for the parameters of the MR stochastic risk premium; they address this uncertainty by fully adopting robust optimization for all model drift parameters. They only consider applications to uncertainty in financial market, however. Our work picks up where they left off, by looking at the implications for insurance-reinsurance-investment problems. Moreover, we use jump-based and diffusive models, as is appropriate for distinguishing between event frequencies in the claims process and the market processes. Thus, based on Yi et al. (2015a), we incorporate market and claim stochasticity as well as modeling uncertainty for both the insurance model and the financial market model, and investigate an optimal robust reinsurance-investment strategy for the insurer, where the appreciation rate of the stocks and any possible arbitrage-inducing mispricing between two stock prices are described by MR processes. We analyze the impact of ambiguity aversion and mispricing on the optimal strategy.

We say a few more words in this introduction about ambiguity and about mispricing, and how our work incorporates these features in the reinsurance-investment problem under time-varying market conditions.

Ambiguity was developed as a way of addressing modeling uncertainty on the mean rates of return and other drift parameters in stochastic models for risky assets. Therefore, ambiguity has an important impact on investment decisions. This idea has been developed systematically as a method in quantitative investment finance for portfolio selection and asset pricing with model uncertainty or model misspecification. For example, Anderson et al. (2000) introduced ambiguity aversion into the Lucas model, and formulated alternative models. Uppal and Wang (2003) extended Anderson et al. (2000), and developed a framework which allows investors to consider the level of ambiguity. Maenhout (2004, 2006) optimized an inter-temporal consumption problem with ambiguity, and derived closed-form expressions for the optimal strategies. The resulting optimal decision schemes are legitimately called *Robust optimization* because they are highly robust to drift misspecifications. However these ideas should not be limited to financial risk modeling, and the same ambiguity exists in the expected surplus of insurers: some scholars have developed decision optimization under ambiguity to discuss the optimal reinsurance and investment problem. For instance, Zhang and Siu (2009) and Korn et al. (2012) used stochastic differential games to study the optimal reinsurance-investment problem with ambiguity; Yi et al. (2013, 2015a, b) studied the optimal proportional reinsurance-investment strategy with ambiguity aversion under expected utility maximization and mean–variance criterion. Li et al. (2017) focused on another kind of reinsurance, the excess-of-loss reinsurance, and discussed the reinsurance-investment problem with ambiguity aversion. Pun and Wong (2015) studied robust investment-reinsurance optimization with multiscale stochastic volatility. Zeng et al. (2016) studied the equilibrium strategy of a robust optimal reinsurance-investment problem under the mean–variance criterion. In all papers mentioned above, the (excess) mean rates of return of the risky assets are assumed to be constant. However, in real-world market as we mentioned, the appreciation rate of the risky asset changes with time and with market conditions.

In order to derive a more realistically implementable strategy, our work incorporates these time-varying conditions into our stochastic models for financial risk by using stochastic risk premia, and allows for mispricing between stocks prices, all the while studying the optimal reinsurance-investment question in the framework of model drift ambiguity and its robust optimal solutions.

Mispricing is a difference (discrepancy) between a pair of asset prices, where these prices describe assets or contingent claims which are identical or nearly identical. The asset/contingent-claim values ought to have the same or close to the same price during a same trading period but in reality they do not have the same price in different financial markets, because of the existence of frictions in markets which are not entirely mature. For a simple example, the stock of Agriculture Bank of China is traded on Chinese stock exchanges (Shanghai, Shenzhen) as shares A, and on the Hong Kong stock exchanges as shares H. Usually, the price of share A is different from that of share H. As explained in Gu et al. (2017), before 2015, China's trading policy did not permit individual traders to

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